

Deep learning for medical image reconstruction

Models, priors and data

Ruud van Sloun

Image reconstruction



Magnetic Resonance Imaging
(faster, low-field etc.)

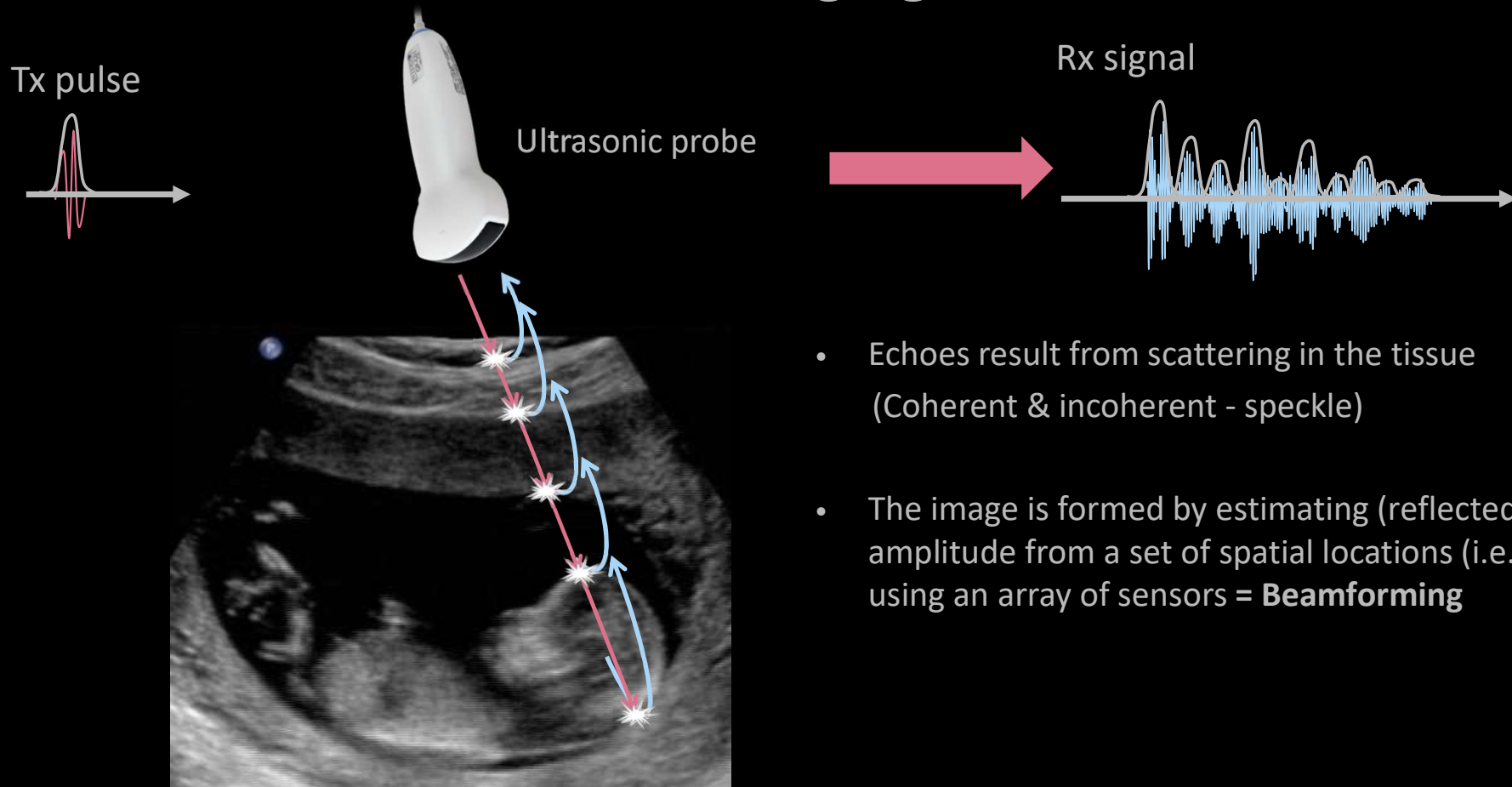


Computed Tomography
(low-dose, sparse-view)



Ultrasound
(high-quality, reduced data-rates)

Ultrasound imaging basics



- Echoes result from scattering in the tissue (Coherent & incoherent - speckle)
- The image is formed by estimating (reflected) signal amplitude from a set of spatial locations (i.e. pixels) using an array of sensors = **Beamforming**

Ultrasound imaging basics

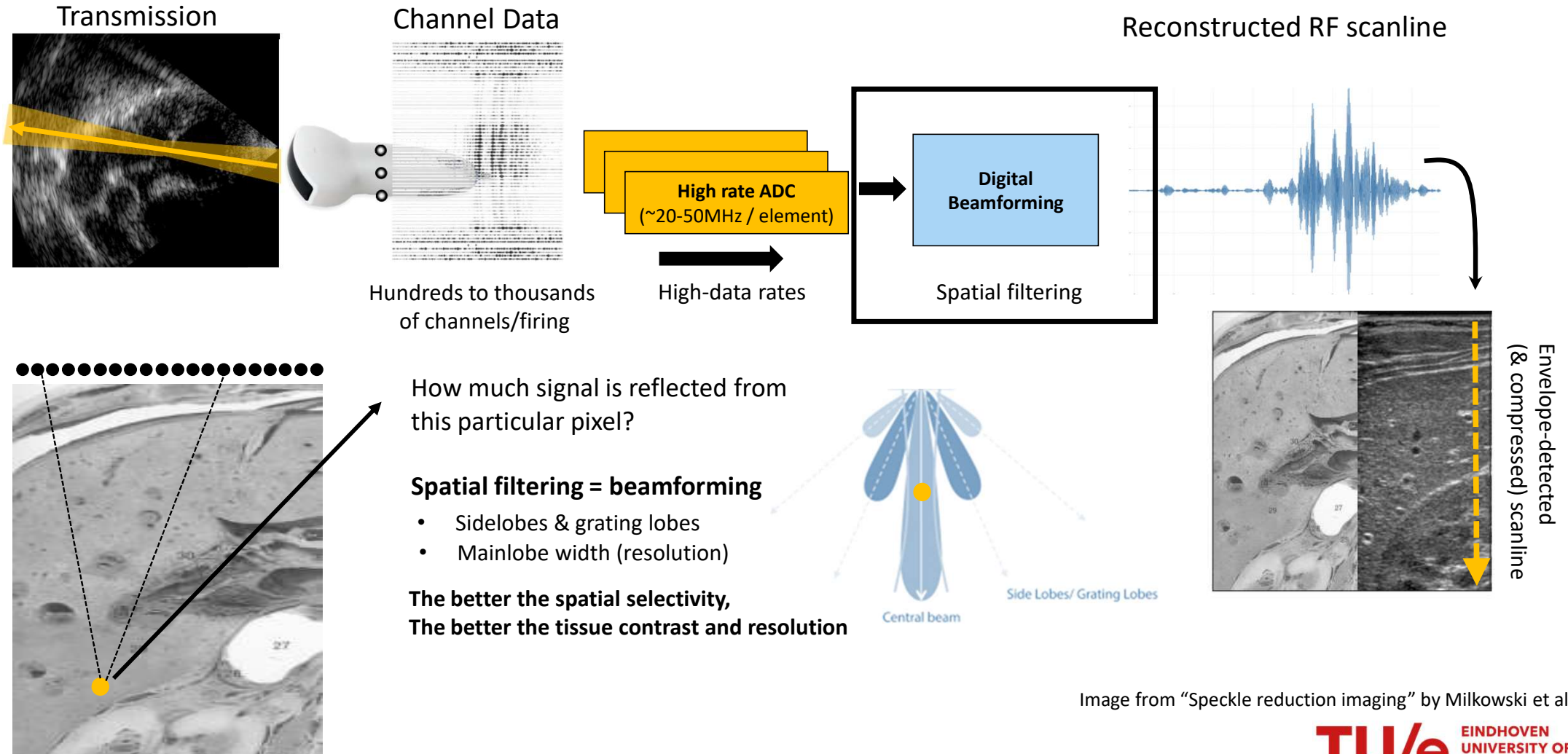
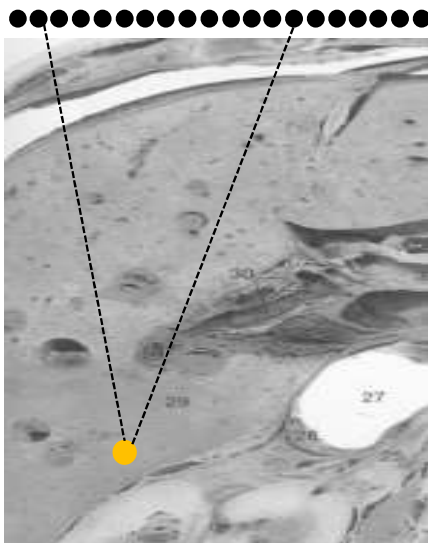


Image from "Speckle reduction imaging" by Milkowski et al.

Ultrasound imaging basics

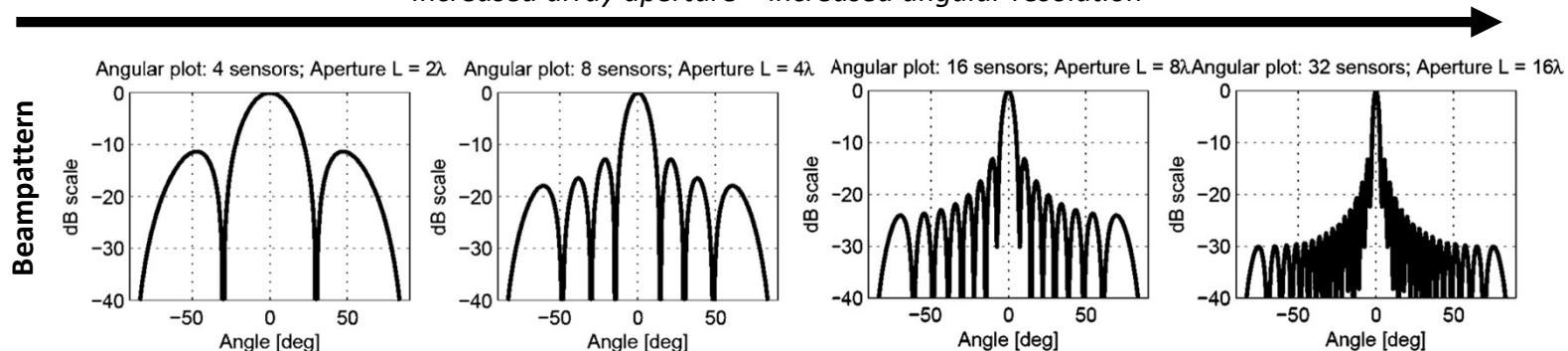


Better spatial selectivity,
Better tissue contrast and resolution

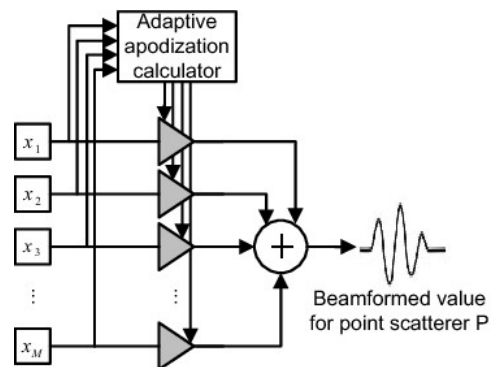
Image quality is a function of:

1. Physics: array geometry and probe bandwidth

Increased array aperture – increased angular resolution



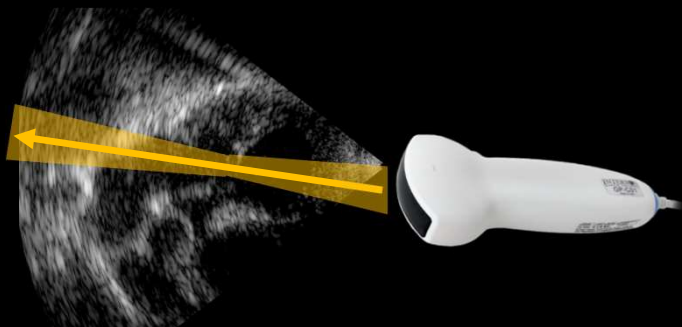
2. Algorithms: powerful digital signal processing and beamforming on RF channel data



➔ AI opportunity

Ultrasound imaging AI opportunities

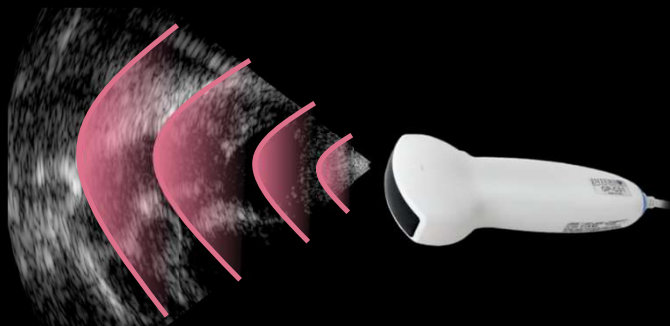
Focused transmit



Parallel ultrafast acquisition

- High time-resolution
- Compromising spatial resolution and contrast
- Relies more heavily on receive spatial filtering/beamforming

Parallel ultrafast transmit



High image quality under minimal data rates

- Improve tissue contrast (*accurate* contrast)
- Resolution depends on array aperture -> high-res with small/sparse aperture?
- Compressed sensing to reduce data rates at the probe

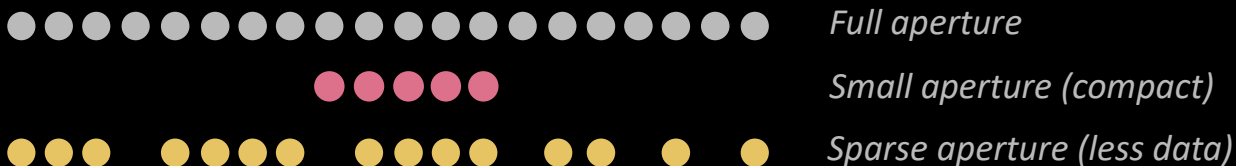
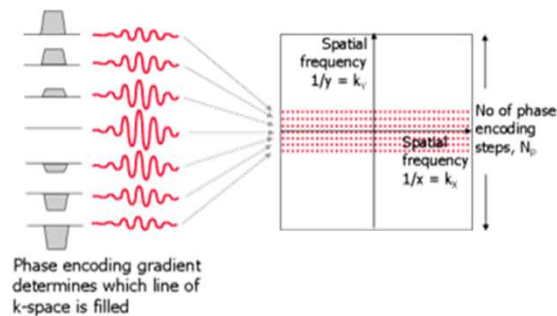
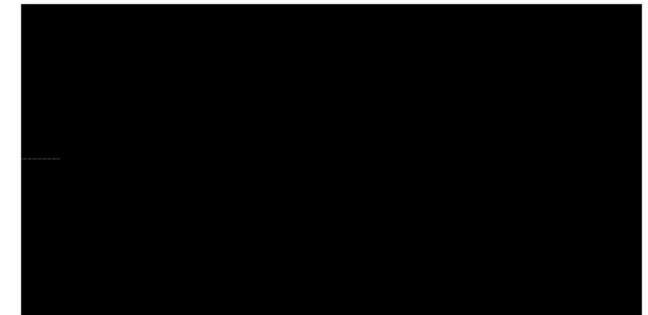
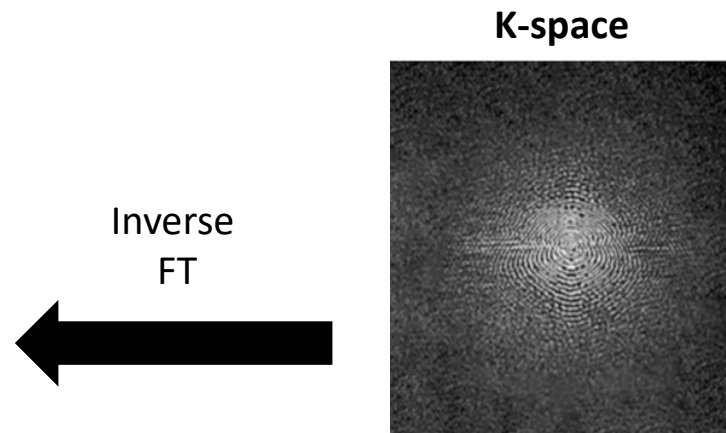


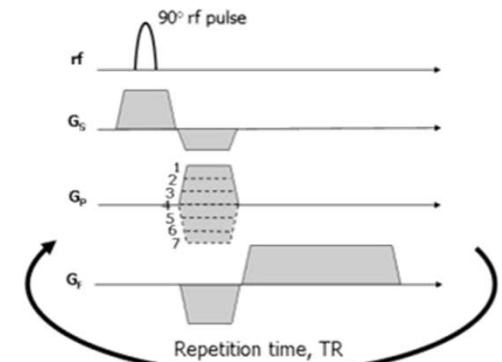
Image basics: MRI



Magnetic Resonance Imaging
(faster, low-field etc.)



Pulse sequence



Total time $\sim TR \times \text{amount of repetitions/lines}$

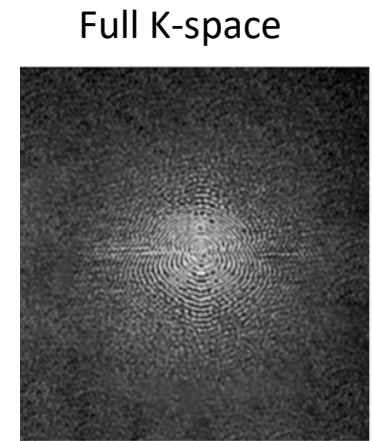
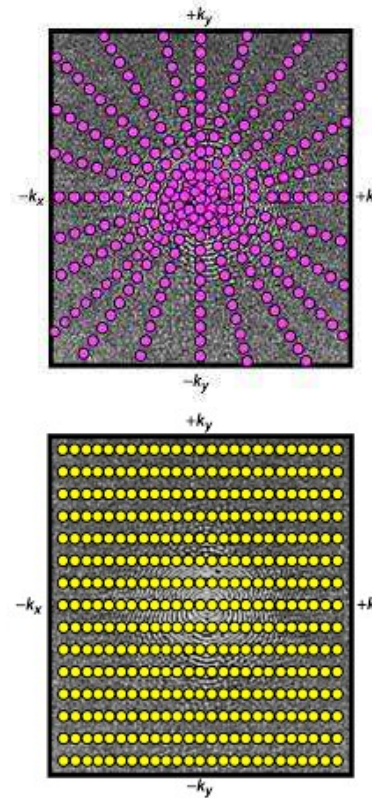
Can we go faster?

Image basics: MRI



Magnetic Resonance Imaging
(faster, low-field etc.)

Undersampled K-space



$p(x|y)?$
Reconstruction
algorithm

Sampling/
acquisition

How?

Priors
 $p(x)?$

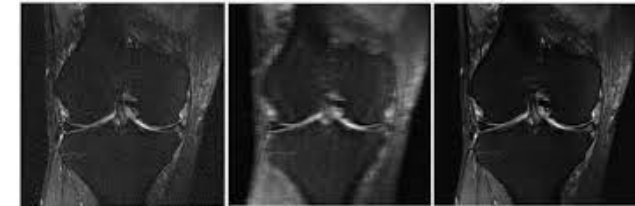
= less acquisitions = faster

$p(y|x)?$

Image reconstruction



Magnetic Resonance Imaging
(faster, low-field etc.)



Fast MRI

- Compressed sensing/fewer acquisitions
- Acceleration
- Aliasing artefacts

Low-field MRI

- Lower field strengths: compact MRI machines
- Low SNR k-space -> less clear high-frequencies -> low resolution

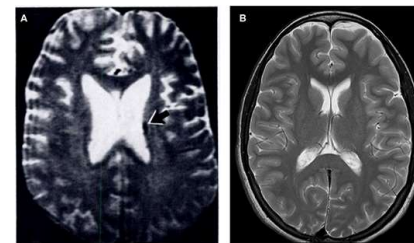


Image reconstruction



Computed Tomography
(low-dose, sparse-view)

Low-dose CT

- Safer – less ionizing radiation
- Low SNR -> limited fidelity

Sparse CT

- reduced scan time and improved time resolution
- Lower dose
- Undersampling artefacts

Low



High



Full



Sparse

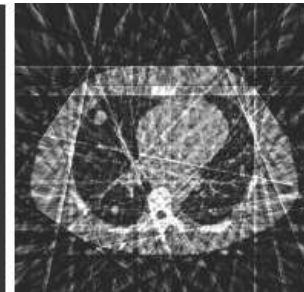


Image reconstruction



Magnetic Resonance Imaging
(faster, low-field etc.)



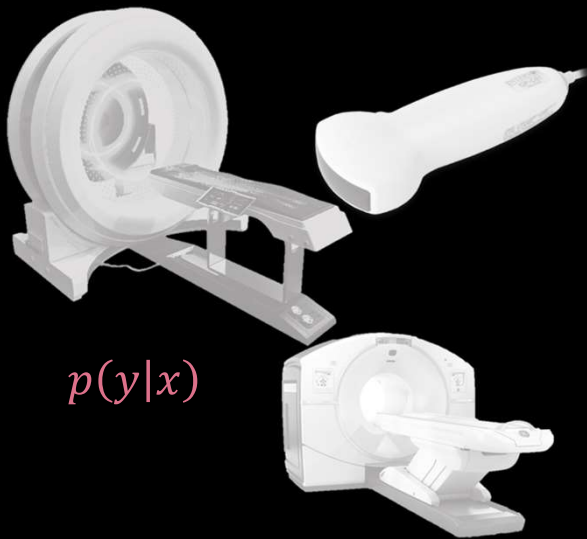
Computed Tomography
(low-dose, sparse-view)



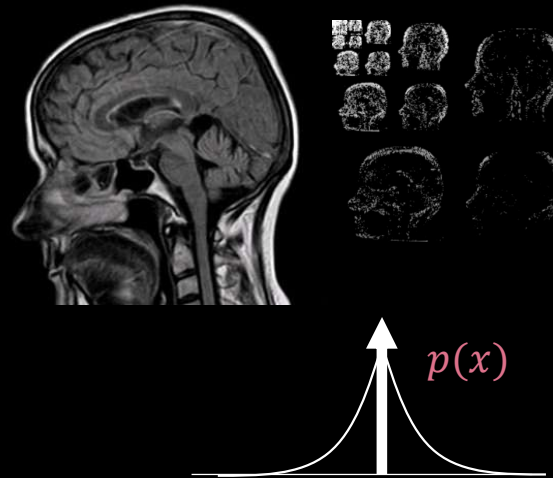
Ultrasound
(high-quality, reduced data-rates)

What gaps can we fill by learning?

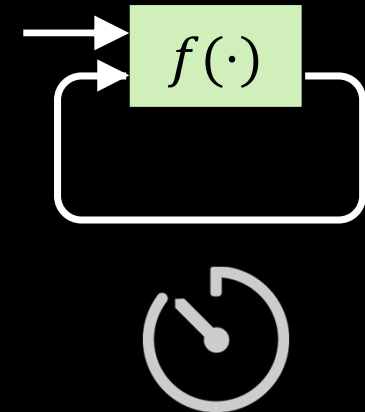
Challenges in classical image recon based on models



Acquisition model assumptions incorrect/imprecise

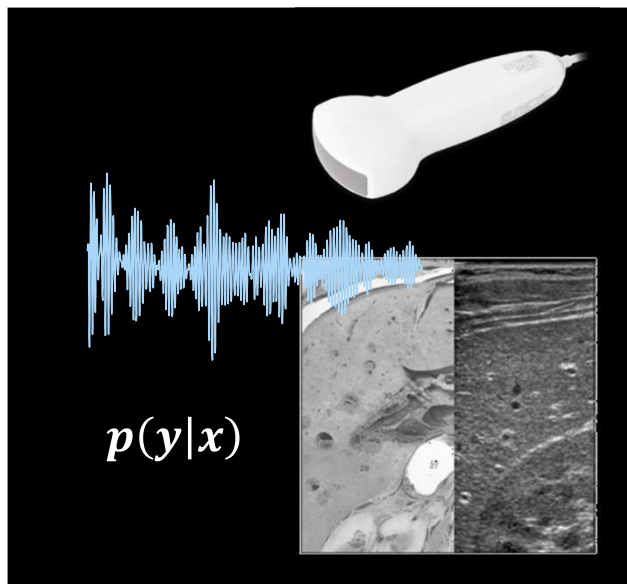


Statistical image priors not sufficiently expressive/accurate



Slow image reconstruction with high complexity (e.g. iterative, matrix inversions etc)

Models and Priors: ultrasound

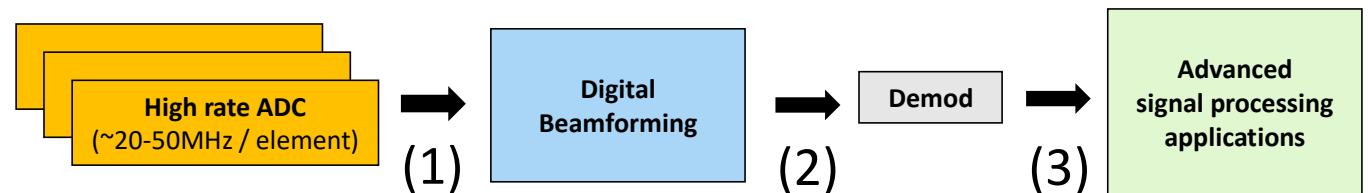


Acquisition model
assumptions incorrect/imprecise
(multiple scattering, aberration
etc.)

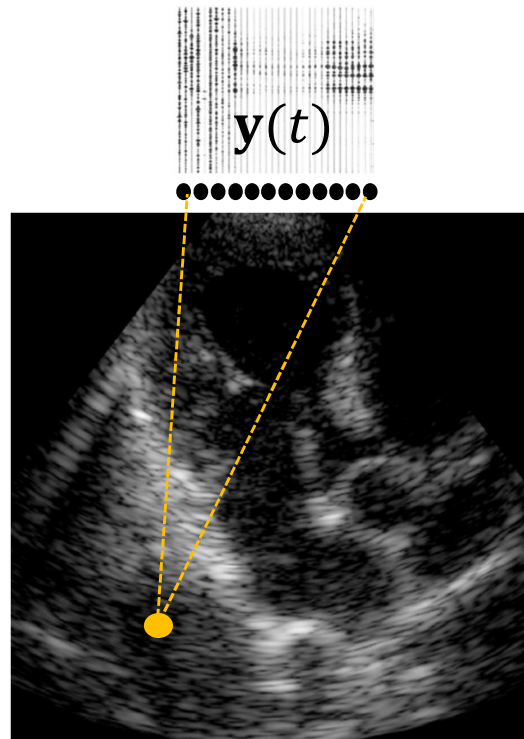
$p(\mathbf{y}|\mathbf{x})$: Likelihood of measurements \mathbf{y} given object \mathbf{x}

Across our imaging pipeline and set of applications,
 \mathbf{y} can be:

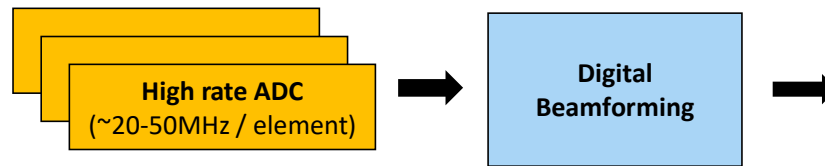
- (1) RF channel data (array response)
- (2) Beamformed RF data
- (3) Image data (beamformed + envelope detected)



Models and Priors: ultrasound



Acquisition model
assumptions incorrect/imprecise
(multiple scattering, aberration
etc.)



Array response (narrowband) single target:

$$\mathbf{y}(t) = \mathbf{a}(\theta)x(t) + \mathbf{n}(t)$$

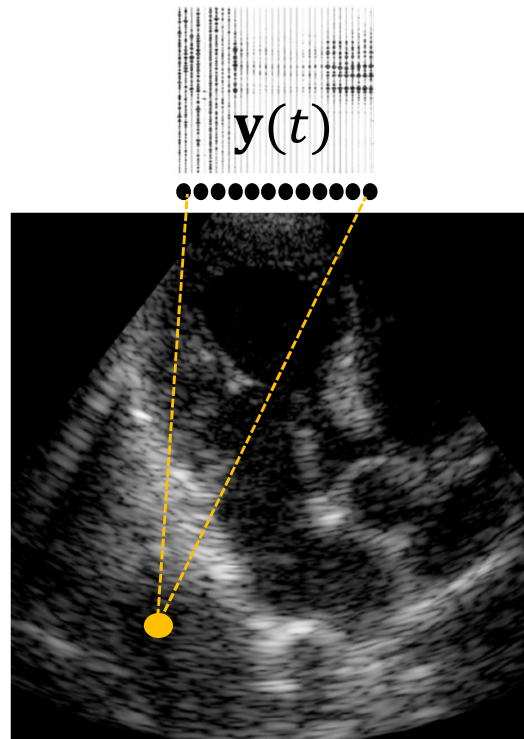
$\mathbf{a}(\theta)$: Array response vector:
 (assumed constant SoS,
 no aberration)

- Noise vector:**
- Sensor noise
 - Off-axis scattering/interference
 - Reverberation
- ➔ In practice complex statistics

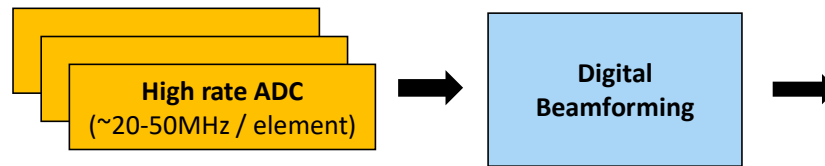
Typical assumption: $\mathbf{n}(t) \sim \mathcal{N}(0, \sigma_n^2 I)$

➔ *MSE-optimal* matched filter: $\hat{x}(t) = \mathbf{w}^H \mathbf{y}(t)$ ($\mathbf{w} = \mathbf{a}$)
 = delay-and-sum (DAS) for wideband

Models and Priors: ultrasound



Acquisition model
assumptions incorrect/imprecise
 (multiple scattering, aberration
 etc.)



Array response (narrowband) single target:

$$\mathbf{y}(t) = \mathbf{a}(\theta)x(t) + \mathbf{n}(t)$$

$\mathbf{a}(\theta)$ → **Array response vector:**
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 no aberration)

$\mathbf{n}(t)$ → **Noise vector:**

- Sensor noise
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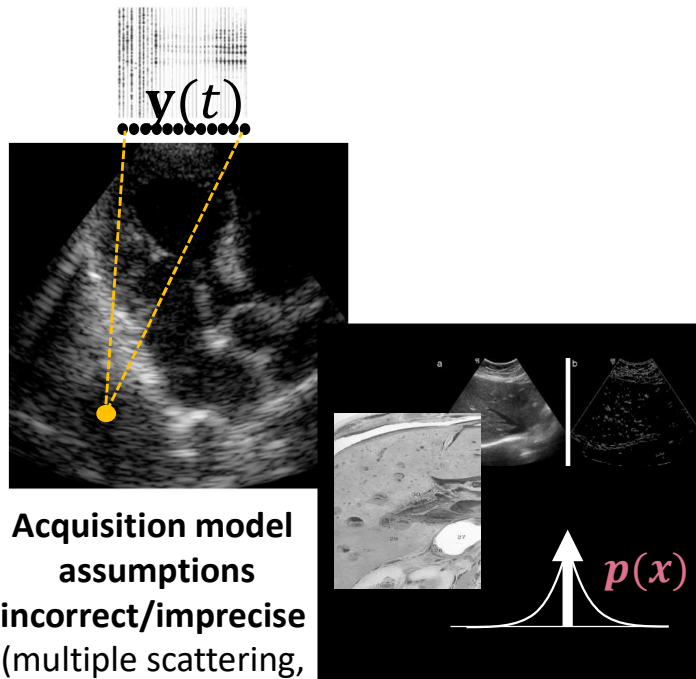
➡ In practice complex statistics

Model-based alternatives to DAS:

- **MVDR** (minimize total (noise) power but retain unity gain)
- **iMAP** (assume $x(t) \sim \mathcal{N}(0, \sigma_x^2)$; iteratively estimate σ_x, σ_n per pixel)
- **ADMIRE** (aperture domain model of signal and noise/interference/clutter; separate using optimization methods)

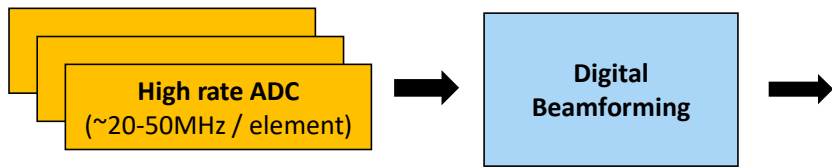
Jensen *et al.*
 Eldar *et al.*
 Byram *et al.*

Models and Priors: ultrasound



Acquisition model assumptions incorrect/imprecise (multiple scattering, aberration etc.)

Statistical image priors not sufficiently expressive/accurate

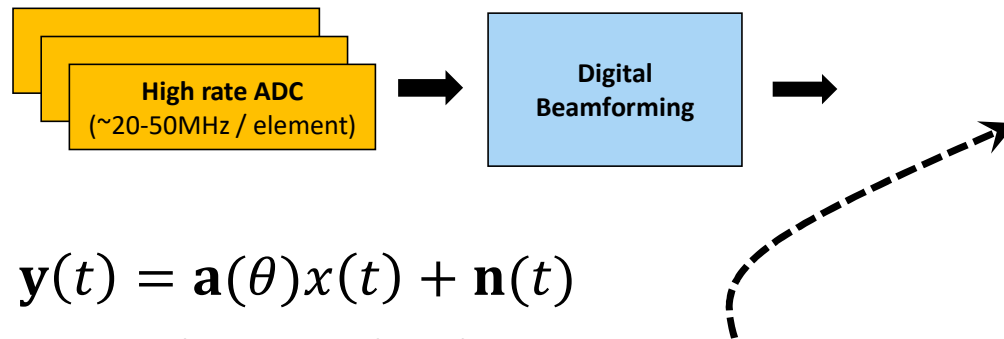
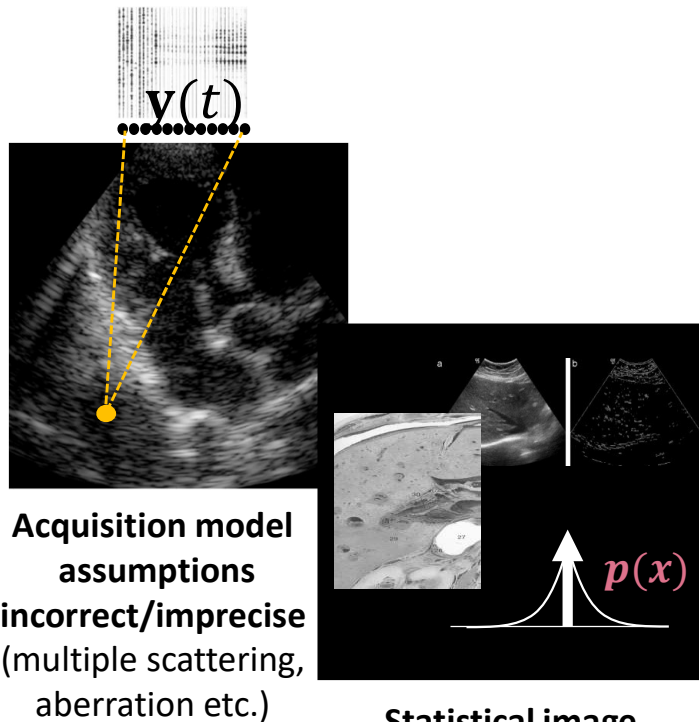


$$\mathbf{y}(t) = \mathbf{a}(\theta)x(t) + \mathbf{n}(t)$$

- **DAS** (matched filter)
- **MVDR** (minimize total (noise) power but retain unity gain)
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General problem: model $p(\mathbf{y}|\mathbf{x})$ too simple / incomplete / or hard to solve due to suboptimal prior $p(\mathbf{x})$

Models and Priors: ultrasound



$$\mathbf{y}(t) = \mathbf{a}(\theta)x(t) + \mathbf{n}(t)$$

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Models and Priors: MRI

$p(\mathbf{y}|\mathbf{x})$: Likelihood of measurements \mathbf{y} given object \mathbf{x}

K-space measurement:

$$\mathbf{y} = \mathbf{A}\mathbf{F}\mathbf{x} + \mathbf{n} \quad \rightarrow \quad \mathbf{A}: \text{Subsampling matrix}$$
$$\mathbf{F}: \text{2D Fourier transform}$$

For low-field: \mathbf{n} is dominating higher frequencies

For fast-MRI: \mathbf{A} is sampling below Nyquist (compressed sensing)

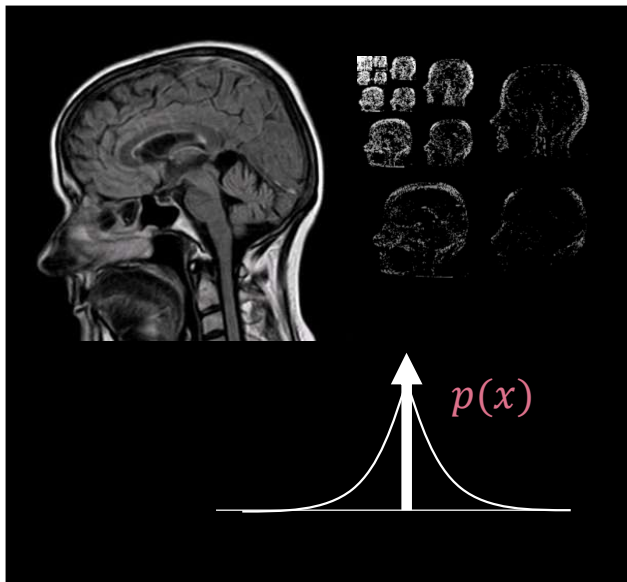
Classical reconstruction: $\mathbf{x} = \mathbf{F}^{-1}\mathbf{z}$ (\mathbf{z} = zero-filled k-space \mathbf{y})

Inverse (MAP) problem:

$$\hat{\mathbf{x}} = \operatorname{argmax}_{\mathbf{x}} p(\mathbf{y}|\mathbf{x})p(\mathbf{x})$$

$$\hat{\mathbf{x}} = \operatorname{argmin}_{\mathbf{x}} \frac{1}{2} \|\mathbf{y} - \mathbf{A}\mathbf{F}\mathbf{x}\|_2^2 - \log p(\mathbf{x})$$

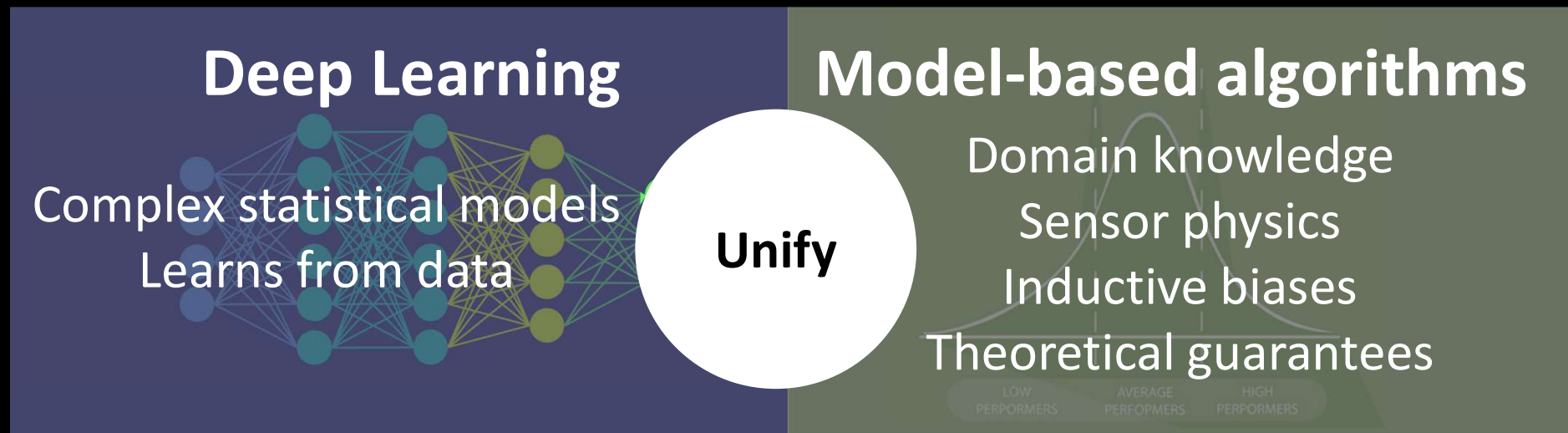
General problem: prior $p(\mathbf{x})$ not sufficiently expressive



Statistical image priors
not sufficiently
expressive/accurate

What if we could improve our models using deep learning without ending up just throwing away what we do know about the problem?

Model-based deep learning



How to bring together to get the best of both?

Van Sloun *et al.* *Proceedings of the IEEE*, 2020

Luijten, ..., Van Sloun. *IEEE trans. med. im.*, 2020

Solomon, ... van Sloun, Eldar. *IEEE trans. med. im.*, 2019

One step back: Model-based optimization

General recipe: construct a model-based optimization algorithm based on a-priori knowledge of the measurement process and statistical priors

Define measurement model with image of interest \mathbf{x} :

$$\mathbf{y} = \mathbf{Ax} + \mathbf{n}$$

Formulate MAP optimization problem:

$$\hat{\mathbf{x}} = \operatorname{argmax}_{\mathbf{x}} p(\mathbf{y}|\mathbf{x}) p(\mathbf{x})$$

$$\hat{\mathbf{x}} = \operatorname{argmin}_{\mathbf{x}} -\log (p(\mathbf{y}|\mathbf{x}) p(\mathbf{x}))$$

Likelihood model under Normal distribution:

$$p(\mathbf{y}|\mathbf{x}) = c e^{-\frac{1}{2}(\mathbf{y}-\mathbf{Ax})^T \boldsymbol{\Sigma}^{-1}(\mathbf{y}-\mathbf{Ax})}$$

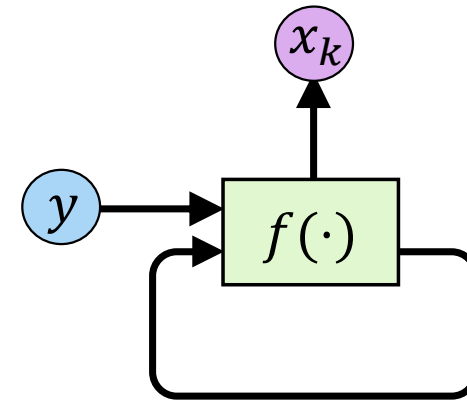
$$\hat{\mathbf{x}} = \operatorname{argmin}_{\mathbf{x}} \frac{1}{2}(\mathbf{y}-\mathbf{Ax})^T \boldsymbol{\Sigma}^{-1}(\mathbf{y}-\mathbf{Ax}) - \log p(\mathbf{x})$$

(assume uncorrelated noise)

$$\hat{\mathbf{x}} = \operatorname{argmin}_{\mathbf{x}} \frac{1}{2} \|\mathbf{y}-\mathbf{Ax}\|_2^2 - \log p(\mathbf{x})$$



Many iterative solvers (for particular choices of $p(\mathbf{x})$)
ISTA, ADMM, etc.



*Iterative model-based algorithm
with input y and output x*

One step back: Model-based optimization

General recipe: construct a model-based optimization algorithm based on a-priori knowledge of the measurement process and statistical priors

Define measurement model with image of interest \mathbf{x} :

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{n}$$

$$\hat{\mathbf{x}} = \operatorname{argmin}_{\mathbf{x}} \frac{1}{2} \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2^2 - \log p(\mathbf{x})$$

$$\hat{\mathbf{x}} = \operatorname{argmin}_{\mathbf{x}} \frac{1}{2} \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2^2 + g_{\theta}(\mathbf{x}) \quad (\text{regularizer } g_{\theta} \text{ parameterized by } \theta)$$

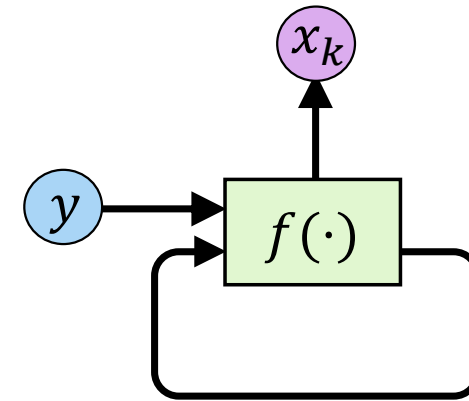
Iterative proximal gradient solvers:

$$\mathbf{z} = \mathbf{x}^k - \mu \left(\nabla_{\mathbf{x}} \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2^2 \right) \Big|_{\mathbf{x}=\mathbf{x}^k} = f_1(\mathbf{x}^k)$$

$$\mathbf{x}^{k+1} = \operatorname{argmin}_{\mathbf{x}} \frac{1}{2} \|\mathbf{z} - \mathbf{x}\|_2^2 + g_{\theta}(\mathbf{x}) = \operatorname{Prox}(\mathbf{z})$$

Prior \rightarrow Proximity function

\rightarrow 2-step factorized optimization



*Iterative model-based algorithm
with input y and output x*

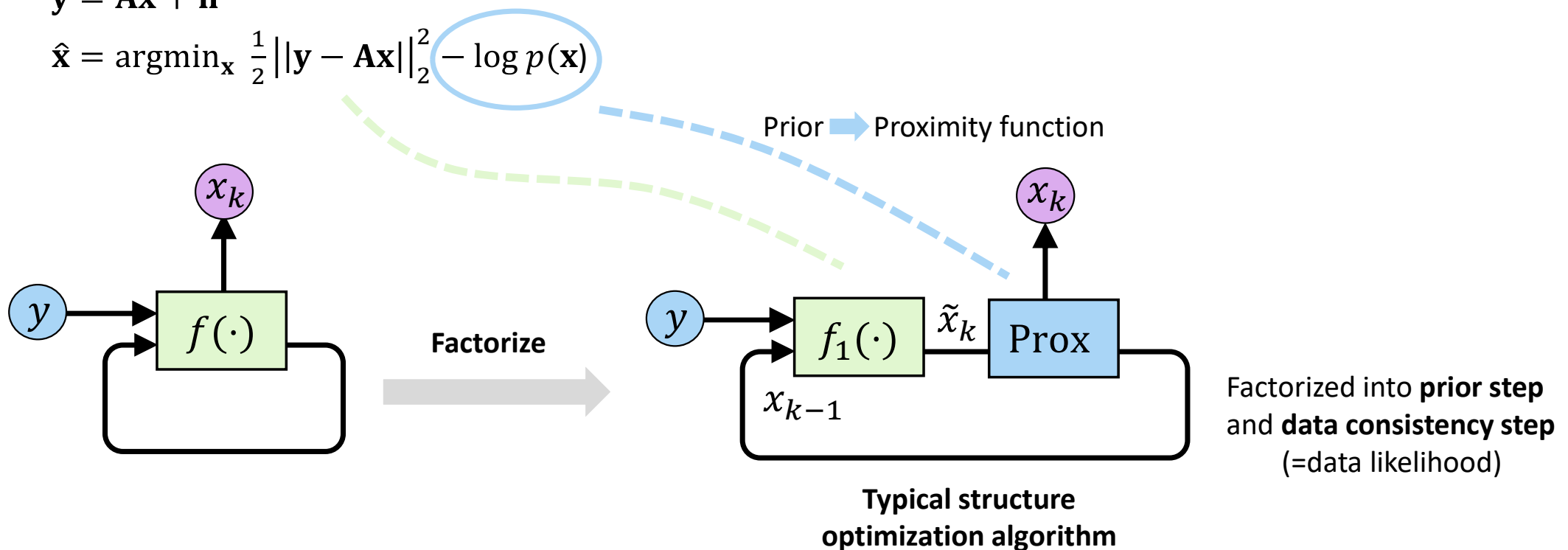
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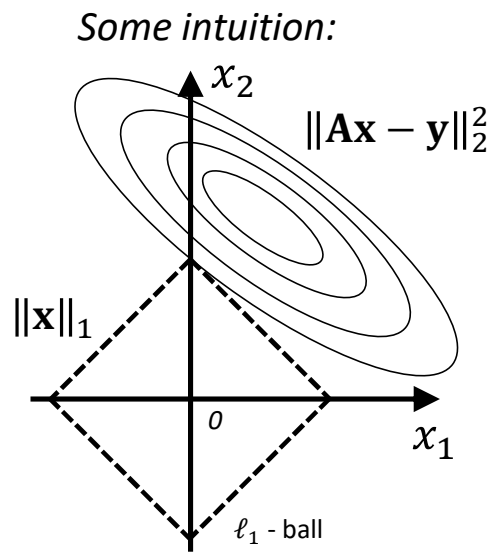
$$\hat{\mathbf{x}} = \operatorname{argmin}_{\mathbf{x}} \frac{1}{2} \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2^2 - \log p(\mathbf{x})$$



One step back: Model-based optimization

Example: sparse coding

Many applications: denoising, compressed sensing, image reconstruction, super-resolution, ...



Sparse coding problem

$$\mathbf{y} = \mathbf{Ax} + \mathbf{n} \quad \text{with } \mathbf{x} \text{ being sparse}$$

MAP problem for \mathbf{x} :

$$\hat{\mathbf{x}} = \underset{\mathbf{x}}{\text{minimize}} \quad \| \mathbf{Ax} - \mathbf{y} \|_2^2 + \lambda \| \mathbf{x} \|_1$$

($\mathbf{x} \sim \text{Laplace}$)

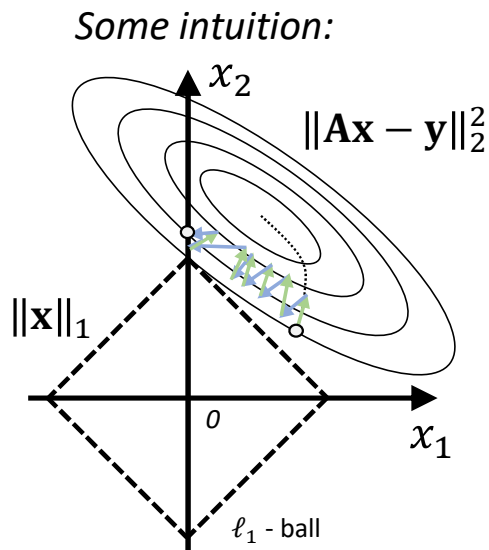


$$-\log p(\mathbf{x}) \sim \| \mathbf{x} \|_1$$

One step back: Model-based optimization

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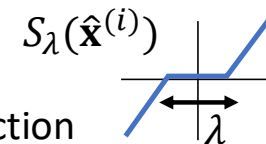
Iterative shrinkage and thresholding (ISTA)

1. Take a gradient step towards $\min_{\mathbf{x}} \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2^2 \rightarrow \mathbf{x} - \mu \nabla_{\mathbf{x}} \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2^2$
2. Move intermediate solution towards prior

→ Solve: $\min_{\mathbf{x}} \|\mathbf{x} - \hat{\mathbf{x}}^{(i)}\|_2^2 + \lambda \|\mathbf{x}\|_1$



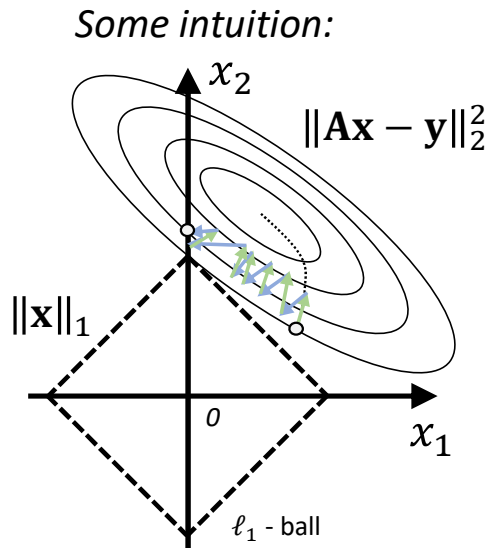
= soft thresholding function



One step back: Model-based optimization

Example: sparse coding

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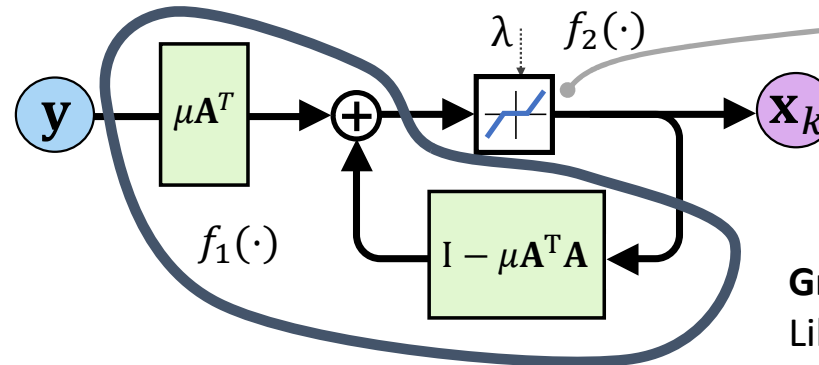
$$\hat{\mathbf{x}} = \underset{\mathbf{x}}{\text{minimize}} \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2^2 + \lambda \|\mathbf{x}\|_1$$

$$(\mathbf{x} \sim \text{Laplace})$$



$$-\log p(\mathbf{x}) \sim \|\mathbf{x}\|_1$$

Iterative shrinkage and thresholding (ISTA)



Proximity operator

Soft thresholding

$$\min_{\mathbf{x}} \|\mathbf{x} - \hat{\mathbf{x}}^{(i)}\|_2^2 + \lambda \|\mathbf{x}\|_1 = S_{\lambda}(\hat{\mathbf{x}}^{(i)})$$

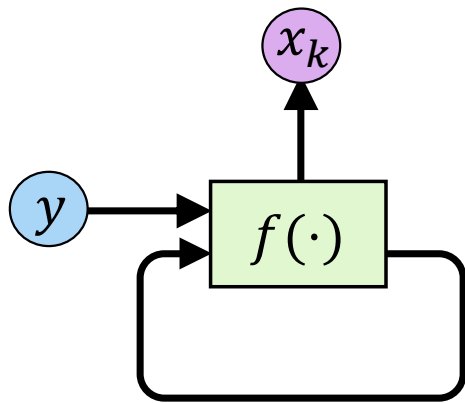
Gradient update step w.r.t.

Likelihood term

$$\mathbf{x} - \mu \nabla_{\mathbf{x}} \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2^2$$

Model-based DL: Deep unfolding

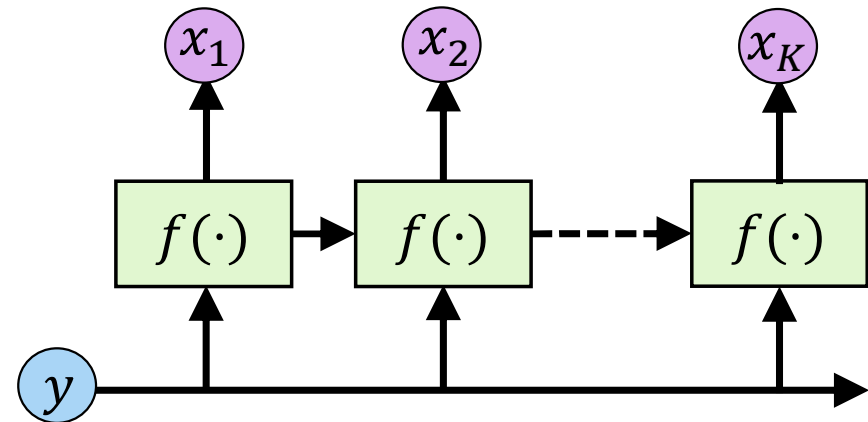
General recipe: take a model-based optimization algorithm, unfold it as a fixed-complexity graph, learn parameters.



*Iterative model-based algorithm
with input y and output x*

**Iterative
algorithm**

“Unfold” K iterations

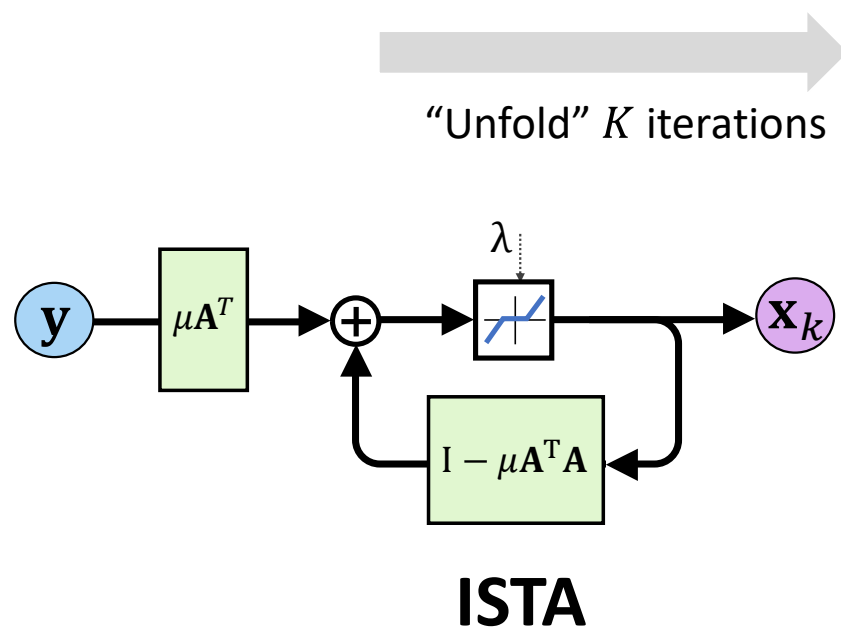


*unfolded model-based algorithm
with input y and output x*

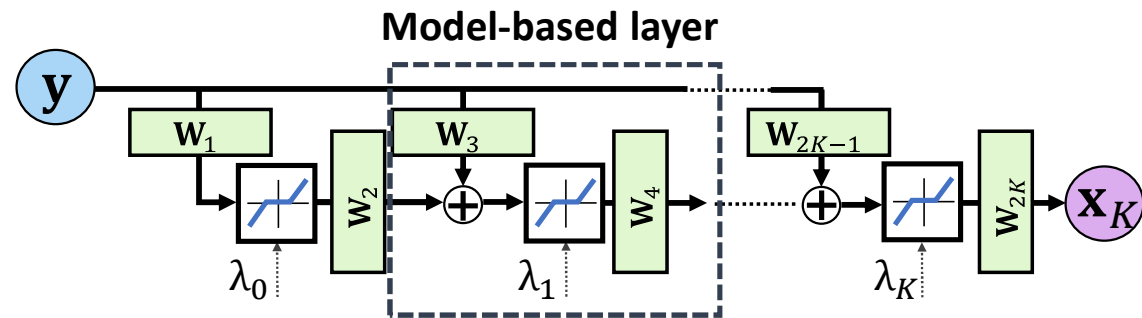
**Learned iterative
algorithm**

Deep unfolding for sparse coding (LISTA)

General recipe: take a model-based optimization algorithm, unfold it as a fixed-complexity graph, learn parameters.



Learned(L)ISTA



Deep learning with a model-based signal prior

- Learn the weight matrices/convolutions W_i
- Learn the thresholding parameters λ_i
- Use the prior (sparsity) and optimization structure

Note: Network nonlinearity (activation function) follows directly from the prior!

Optimization with “plug and play priors”

General recipe: take a model-based optimization algorithm, what if we don't know the prior, or it is complex to describe/model?

Define measurement model with image of interest \mathbf{x} :

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{n}$$

$$\hat{\mathbf{x}} = \operatorname{argmin}_{\mathbf{x}} \frac{1}{2} \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2^2 - \log p(\mathbf{x})$$

$$\hat{\mathbf{x}} = \operatorname{argmin}_{\mathbf{x}} \frac{1}{2} \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2^2 + f_{\theta}(\mathbf{x}) \quad (\text{regularizer } f_{\theta} \text{ parameterized by } \theta)$$

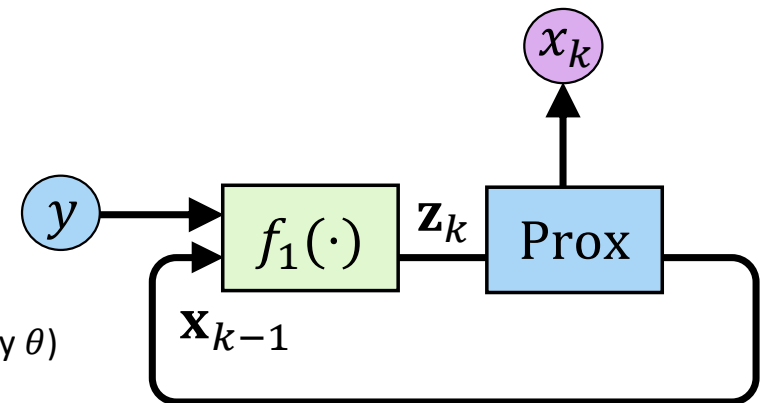
Iterative proximal gradient methods:

$$\mathbf{z} = \mathbf{x}^k - \mu \left(\nabla_{\mathbf{x}} \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2^2 \right) \Big|_{\mathbf{x}=\mathbf{x}^k} = f_1(\mathbf{x}^k)$$

$$\mathbf{x}^{k+1} = \operatorname{argmin}_{\mathbf{x}} \frac{1}{2} \|\mathbf{z} - \mathbf{x}\|_2^2 + f_{\theta}(\mathbf{x}) = \operatorname{Prox}(\mathbf{z})$$

2-step factorized optimization

Prior \rightarrow Proximity function



Typical structure optimization algorithm

Question: what is a good general choice for $\operatorname{Prox}(\mathbf{z})$ in structured signals if you don't know $p(\mathbf{x})$?

A high-performant denoiser

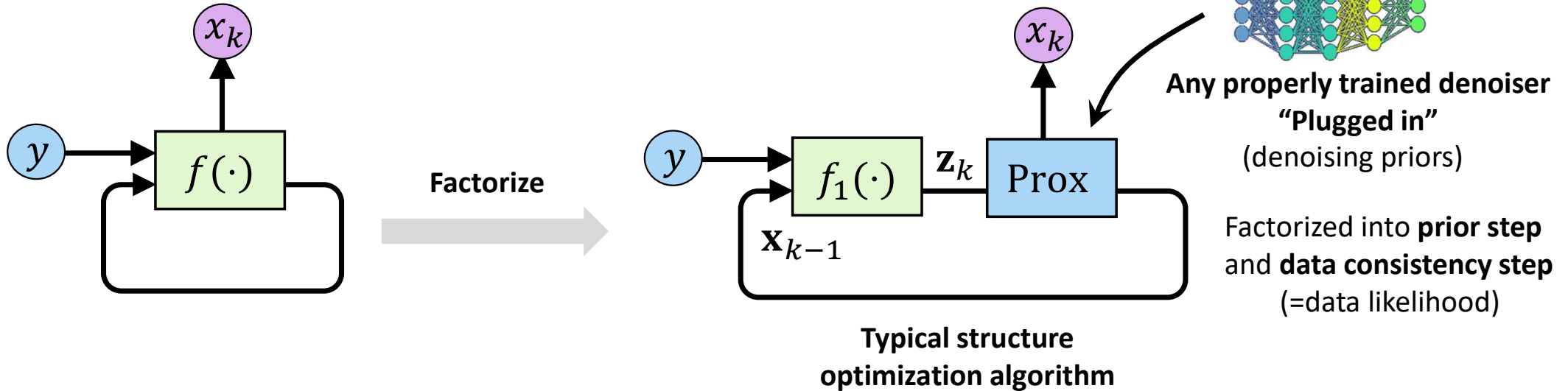
Optimization with “plug and play priors”

General recipe: take a model-based optimization algorithm, “plug in” a high-performant denoiser (e.g. trained deep neural network) as the Prox

Define measurement model with image of interest \mathbf{x} :

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{n}$$

$$\hat{\mathbf{x}} = \operatorname{argmin}_{\mathbf{x}} \frac{1}{2} \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2^2 - \log p(\mathbf{x})$$



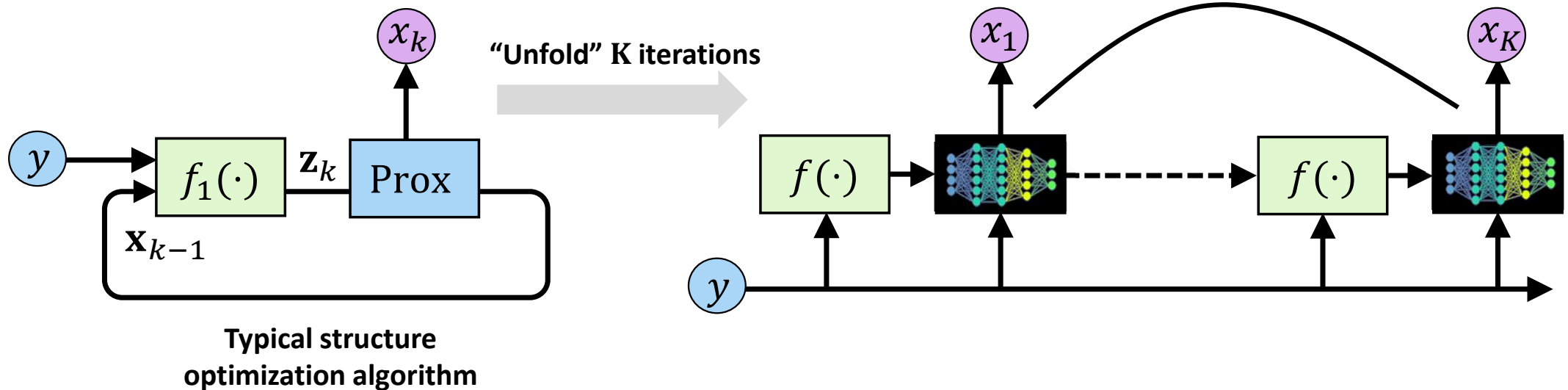
Deep unfolding & end-to-end training

General recipe: take a model-based optimization algorithm, unfold it as a fixed-complexity graph, learn proximal parameters.

Define measurement model with image of interest \mathbf{x} :

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{n}$$

$$\hat{\mathbf{x}} = \operatorname{argmin}_{\mathbf{x}} \frac{1}{2} \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2^2 - \log p(\mathbf{x})$$



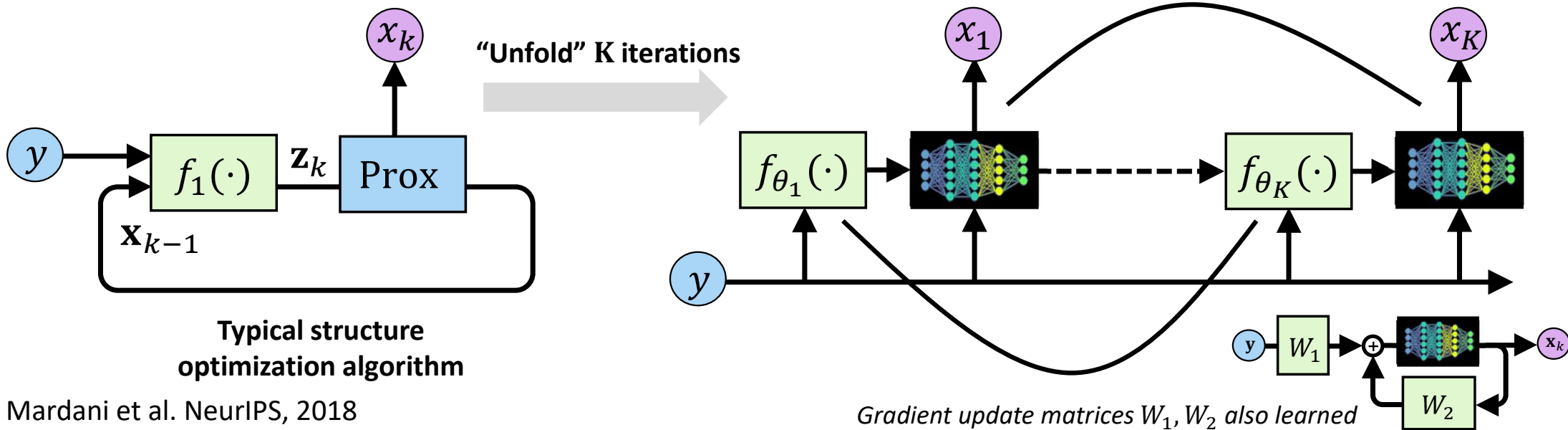
Deep unfolding & end-to-end training

General recipe: take a model-based optimization algorithm, unfold it as a fixed-complexity graph, learn all parameters.

Define measurement model with image of interest \mathbf{x} :

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{n}$$

$$\hat{\mathbf{x}} = \operatorname{argmin}_{\mathbf{x}} \frac{1}{2} \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2^2 - \log p(\mathbf{x})$$



Combining models, priors and deep learning for image reconstruction: **use-cases**



Magnetic Resonance Imaging
(faster, low-field etc.)



Computed Tomography
(low-dose, sparse-view)



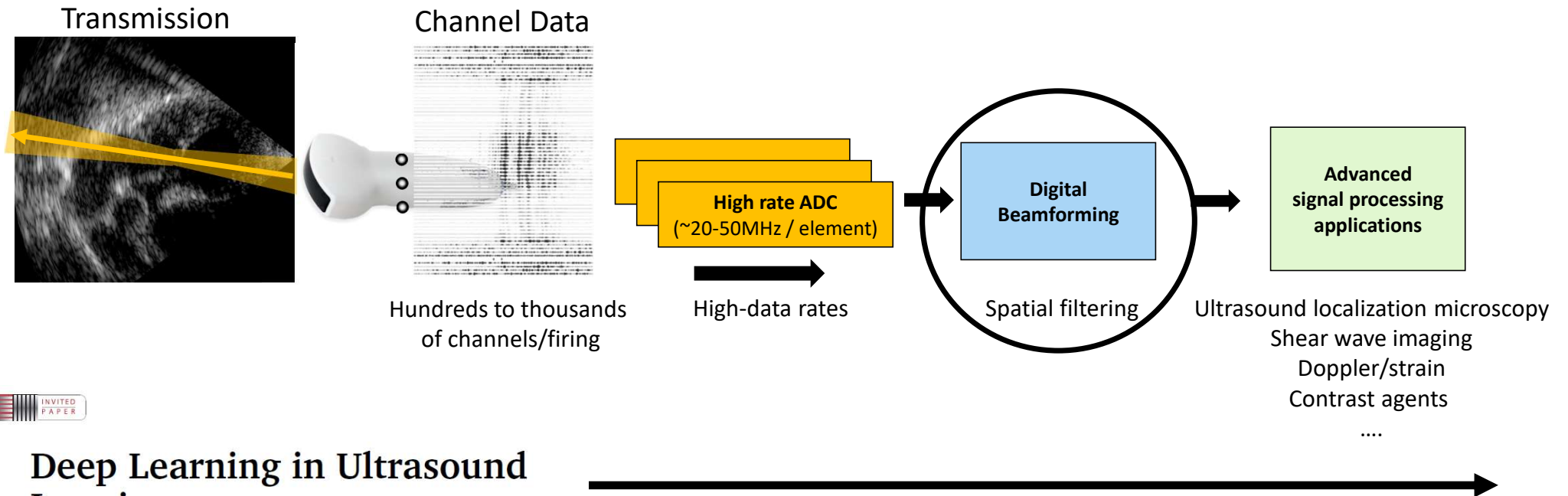
Ultrasound
(high-quality, reduced data-rates)

Combining models, priors and deep learning for image reconstruction: **use-cases**



Ultrasound
(high-quality, reduced data-rates)

Opportunities for ultrasound



Deep Learning in Ultrasound Imaging

Deep learning is taking an ever more prominent role in medical imaging. This article discusses applications of this powerful approach in ultrasound imaging systems along with domain-specific opportunities and challenges.

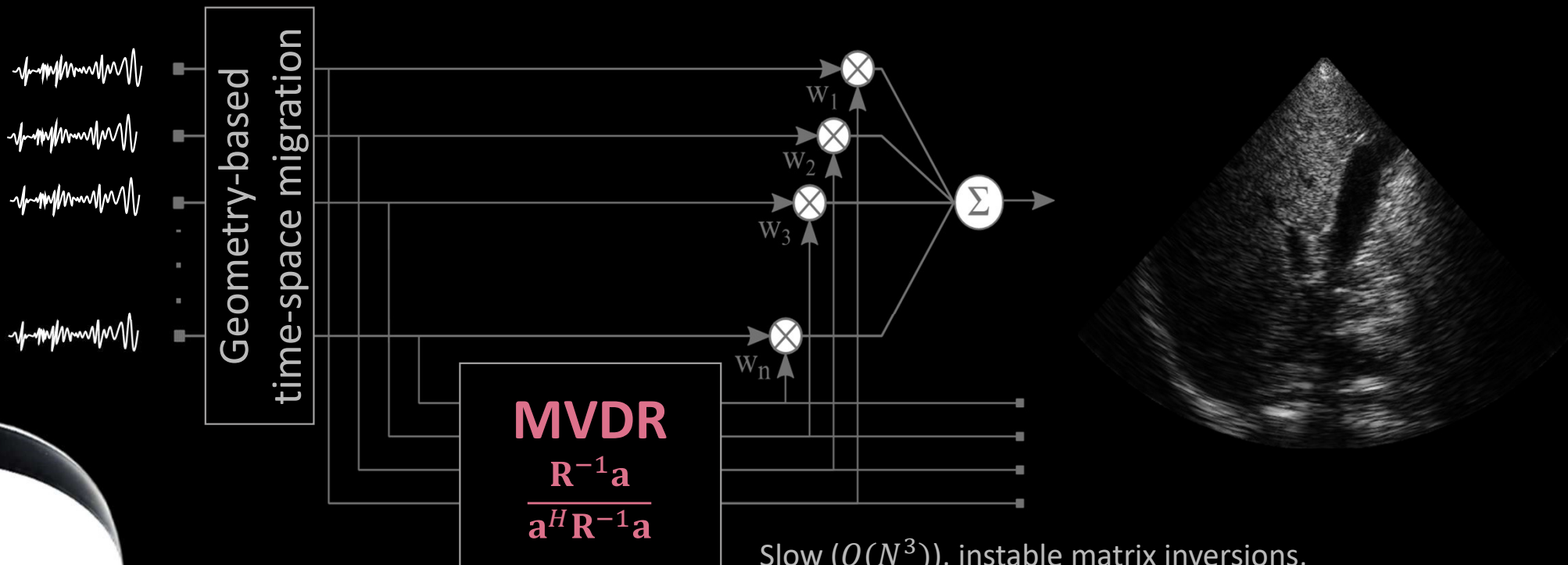
By RUUD J. G. VAN SLOUN¹, Member IEEE, REGEV COHEN, Graduate Student Member IEEE, AND YONINA C. ELДАР², Fellow IEEE

ABSTRACT | In this article, we consider deep learning strategies in ultrasound systems, from the front end to advanced applications. Our goal is to provide the reader with a broad understanding of the possible impact of deep learning methodologies on many aspects of ultrasound imaging. In particular, we discuss methods that lie at the interface of signal acquisition and machine learning, exploiting both data structure (e.g., sparsity in some domains) and data dimensionality (this data staging, and management, as well as for treatment choice, planning, guidance, and follow-up. Among the diagnostic imaging options, ultrasound imaging [1] is uniquely positioned, being a highly cost-effective modality that offers the clinician an unmatched and invaluable level of interaction, enabled by its real-time nature. Its portability and cost effectiveness permit point-of-care imaging at the bedside, in emergency settings, rural clinics, and

Van Sloun, Cohen, Eldar, Proceedings of the IEEE, 2019

Adaptive beamforming by MVDR

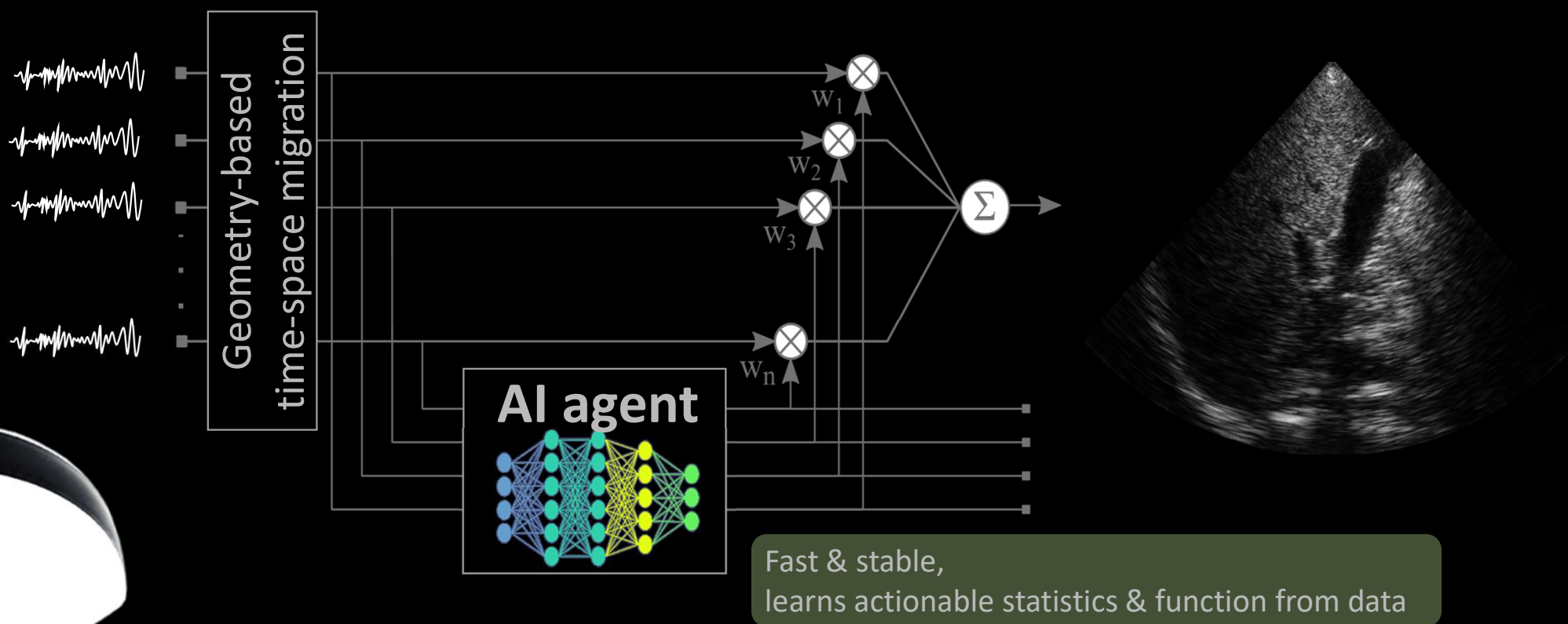
Geometry-based time-space migration + model-based adaptive apodization



Slow ($O(N^3)$), unstable matrix inversions, relies on accurate estimates of statistics (model knowledge)

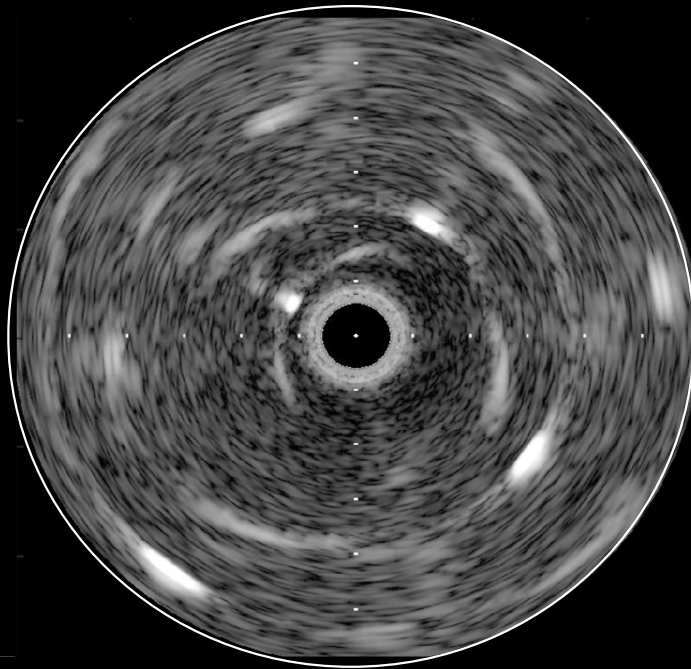
Adaptive beamforming by deep learning (ABLE)

Hybrid inference: Model-based computational graph with integrated NN

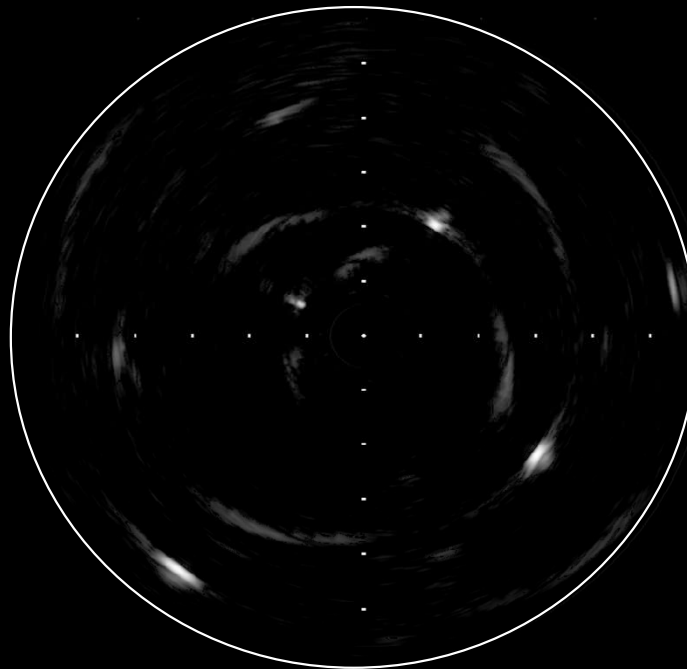


Adaptive beamforming by deep learning (ABLE)

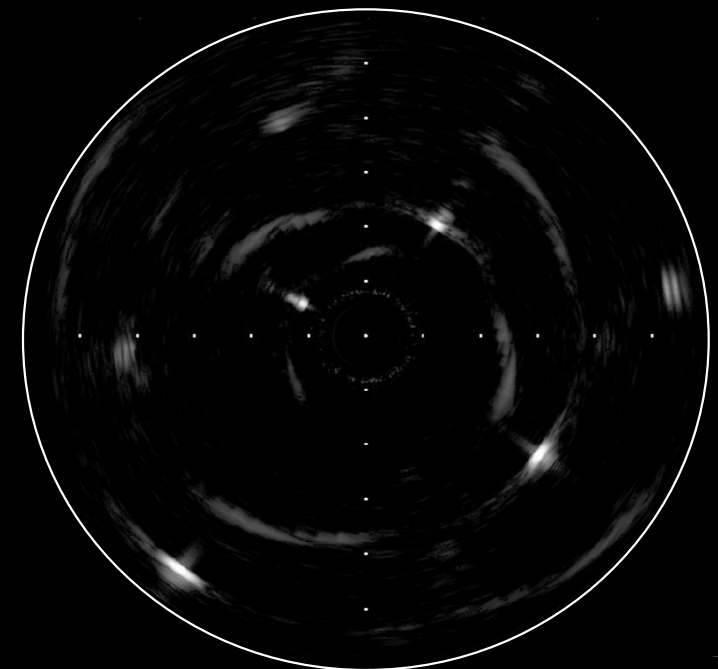
Delay-and-sum (standard)



Deep learning (ABLE)

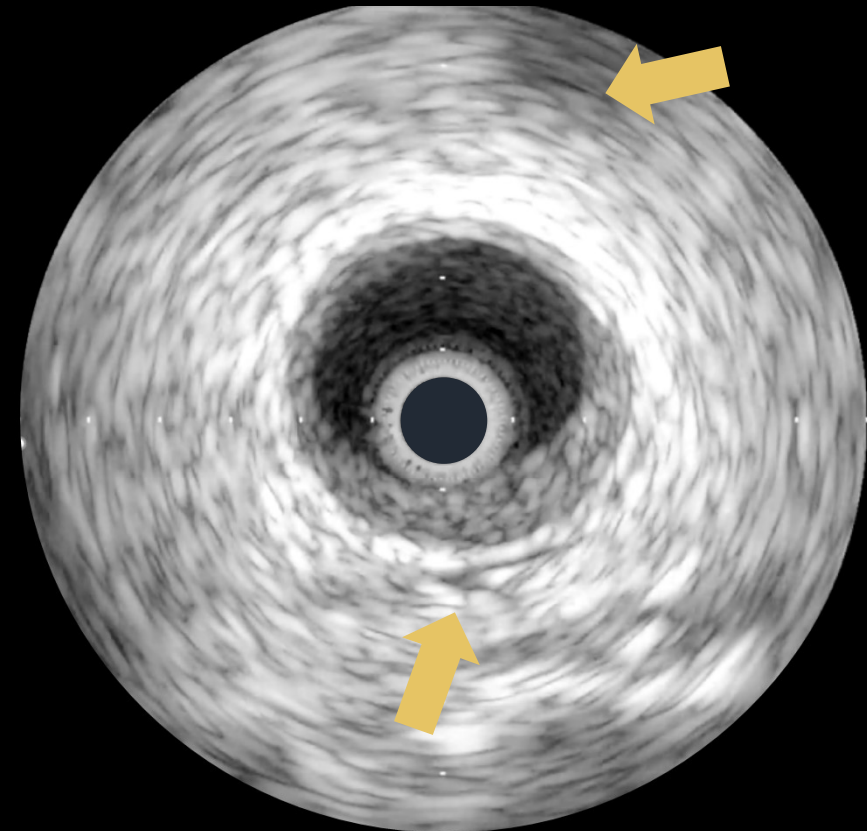


Target (MVDR)

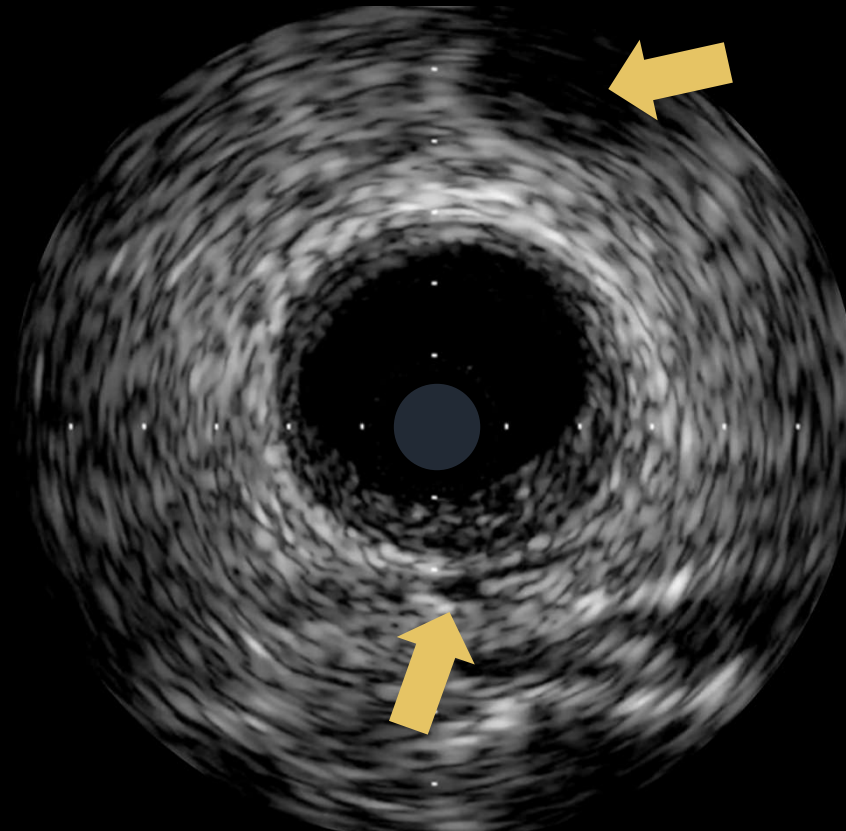


Note: without post-processing and s-curve
60 dB, log-scale

Adaptive beamforming by deep learning (ABLE)



Standard



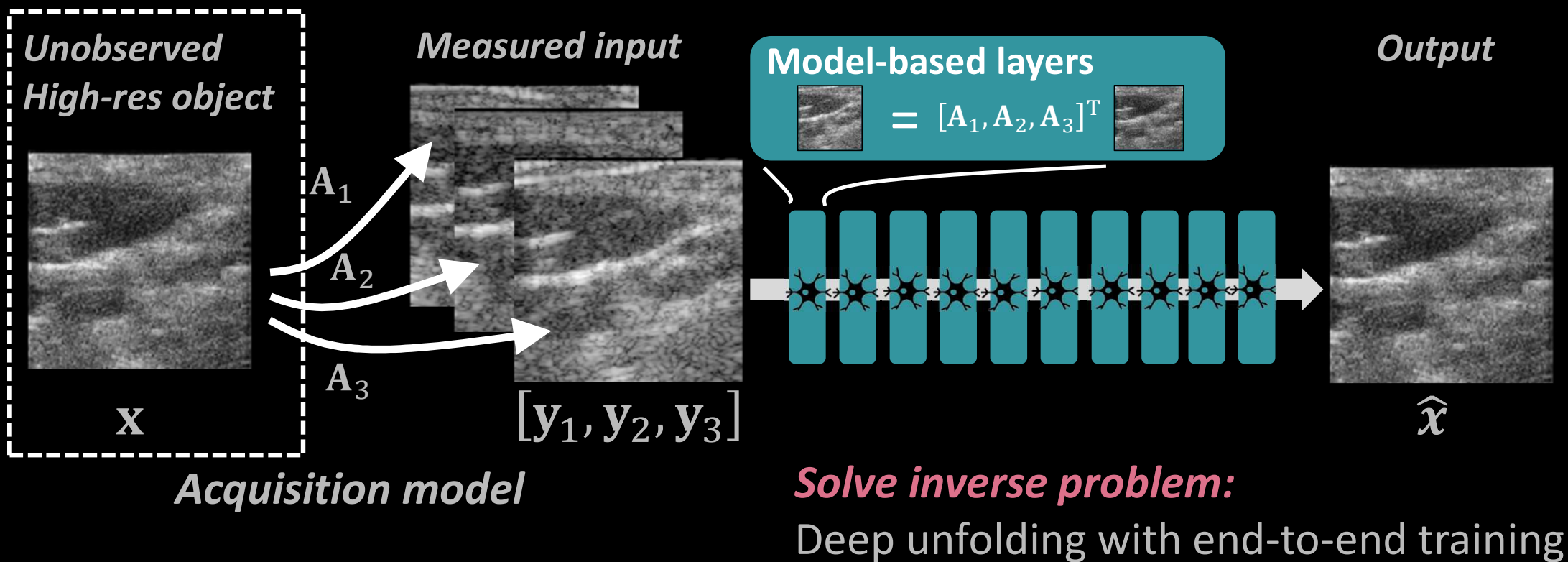
Deep learning (ABLE)

Less clutter
Higher resolution

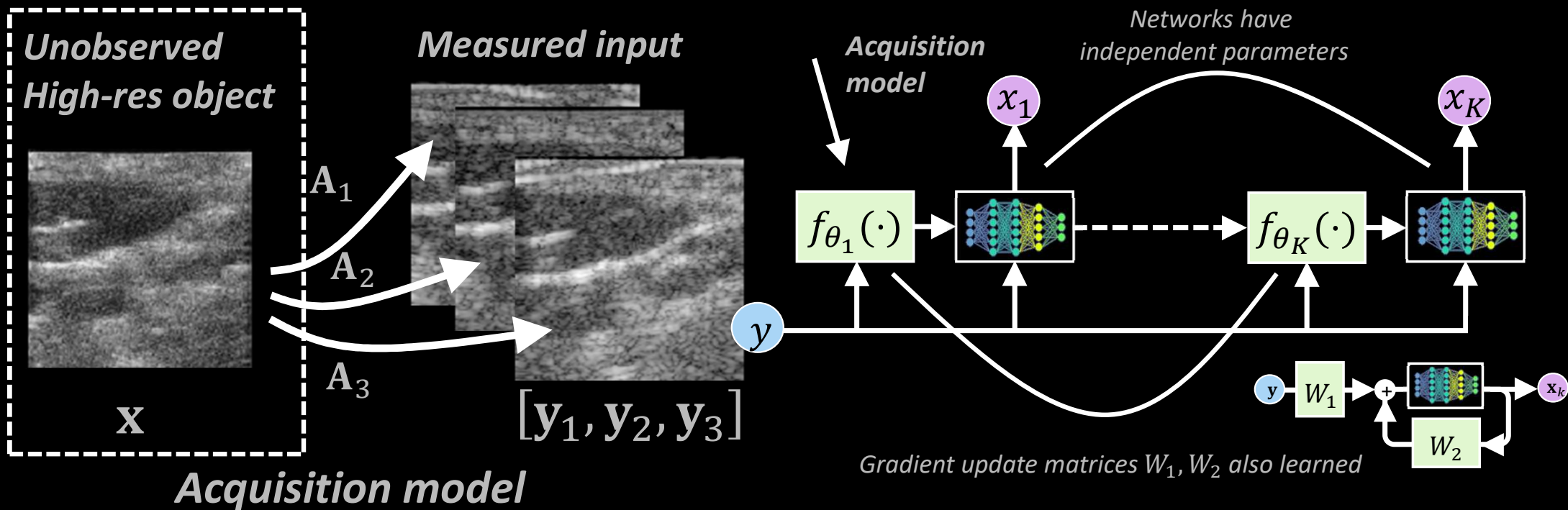
High processing rates,
and robustness

Note: without post-processing and s-curve
60 dB, log-scale

High-resolution plane-wave compounding

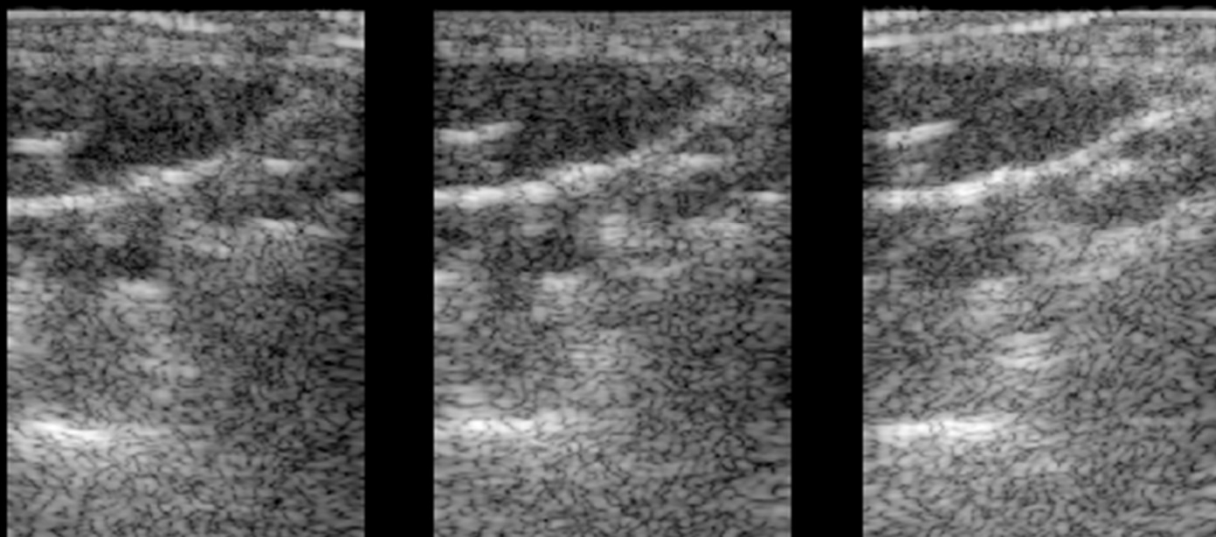


High-resolution plane-wave compounding

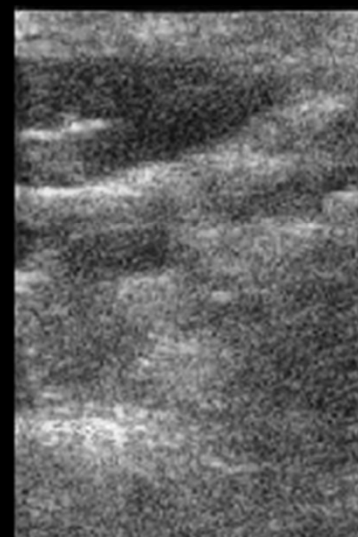


High-resolution plane-wave compounding

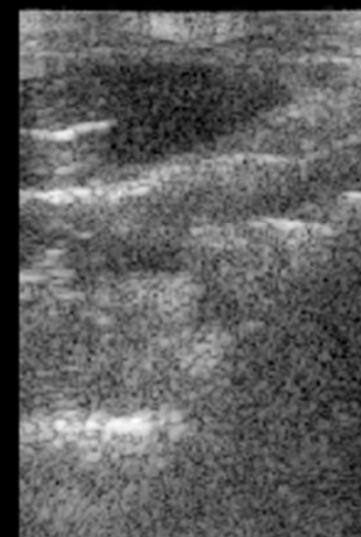
Input images



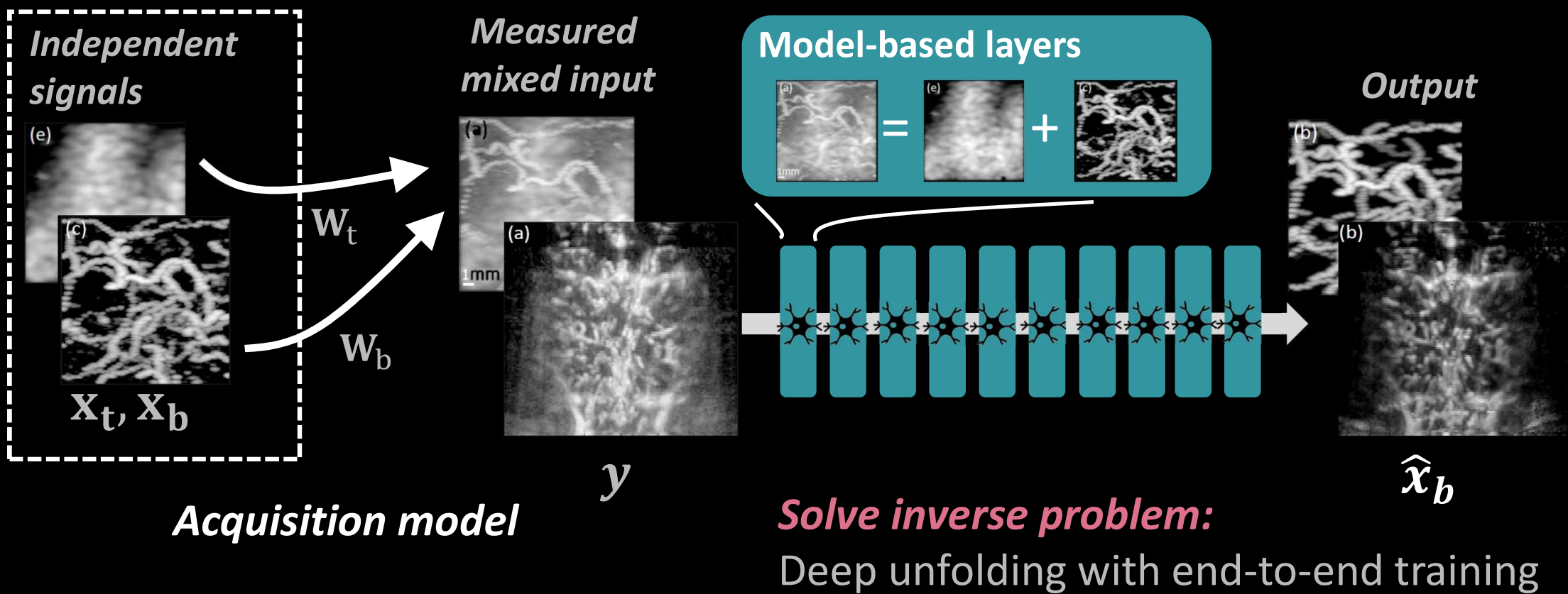
DL output



Target

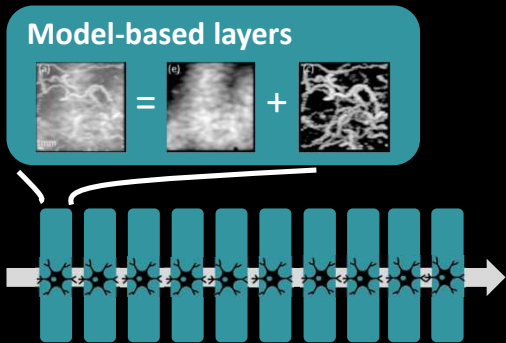


Spatiotemporal source-extraction/dehazing



Solomon et al. *IEEE trans. Med. Imag.*, 2019
van Sloun et al. *Proceedings of the IEEE*, 2020

Spatiotemporal source-extraction/dehazing



Optimization problem:

$$\min_{\mathbf{L}, \mathbf{S}} \frac{1}{2} \|\mathbf{D} - \mathbf{L} - \mathbf{S}\|_F^2 + \lambda_1 \|\mathbf{L}\|_* + \lambda_2 \|\mathbf{S}\|_{1,2}$$

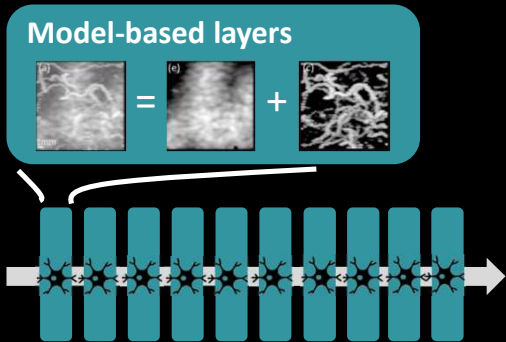
**Prox-grad solution:
(iterative)**

$$\mathbf{L}^{k+1} = \mathcal{SVT}_{\lambda_1/2} \left(\frac{1}{2} \mathbf{L}^k - \mathbf{S}^k + \mathbf{D} \right)$$

$$\mathbf{S}^{k+1} = \mathcal{T}_{\lambda_2/2} \left(\frac{1}{2} \mathbf{S}^k - \mathbf{L}^k + \mathbf{D} \right)$$

Solomon et al. *IEEE trans. Med. Imag.*, 2019
 van Sloun et al. *Proceedings of the IEEE*, 2020

Spatiotemporal source-extraction/dehazing



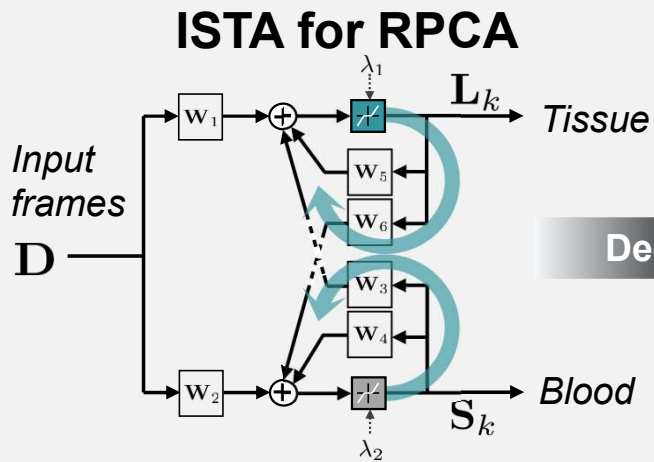
Optimization problem:

$$\min_{\mathbf{L}, \mathbf{S}} \frac{1}{2} \|\mathbf{D} - \mathbf{L} - \mathbf{S}\|_F^2 + \lambda_1 \|\mathbf{L}\|_* + \lambda_2 \|\mathbf{S}\|_{1,2}$$

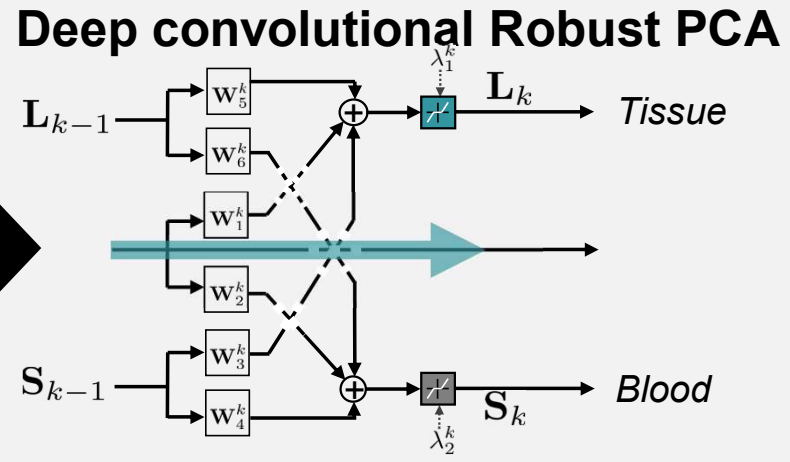
Prox-grad solution:
(iterative)

$$\mathbf{L}^{k+1} = \mathcal{SVT}_{\lambda_1/2} \left(\frac{1}{2} \mathbf{L}^k - \mathbf{S}^k + \mathbf{D} \right)$$

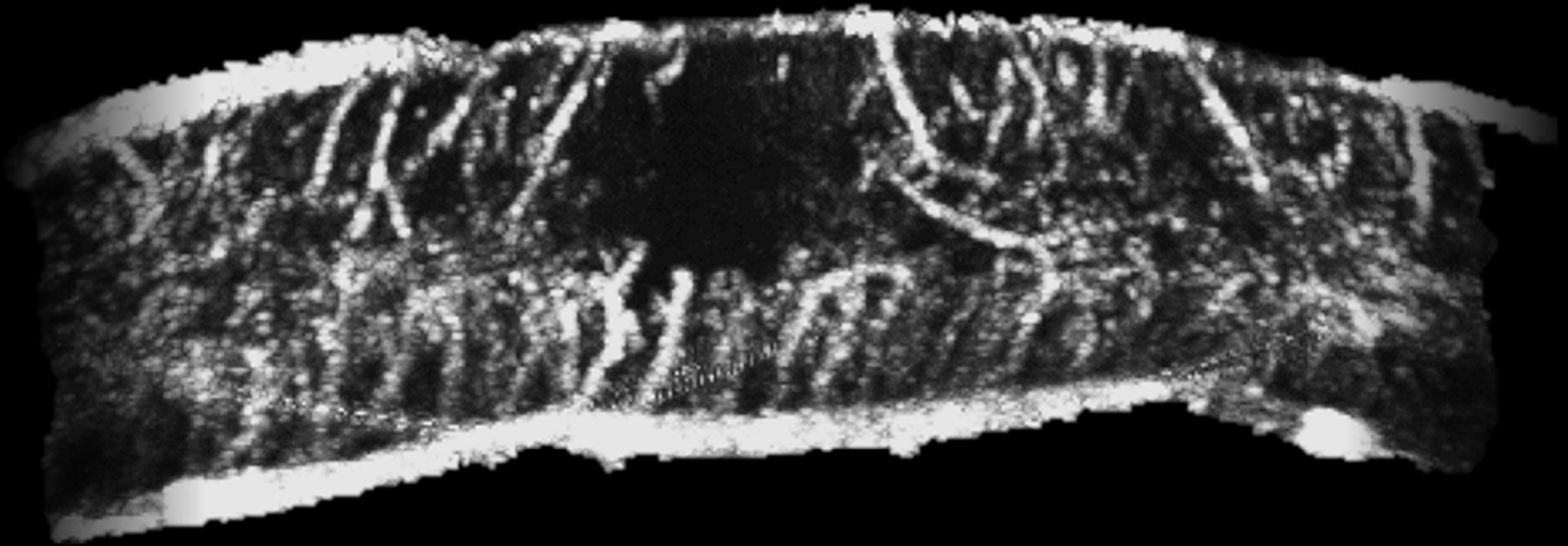
$$\mathbf{S}^{k+1} = \mathcal{T}_{\lambda_2/2} \left(\frac{1}{2} \mathbf{S}^k - \mathbf{L}^k + \mathbf{D} \right)$$



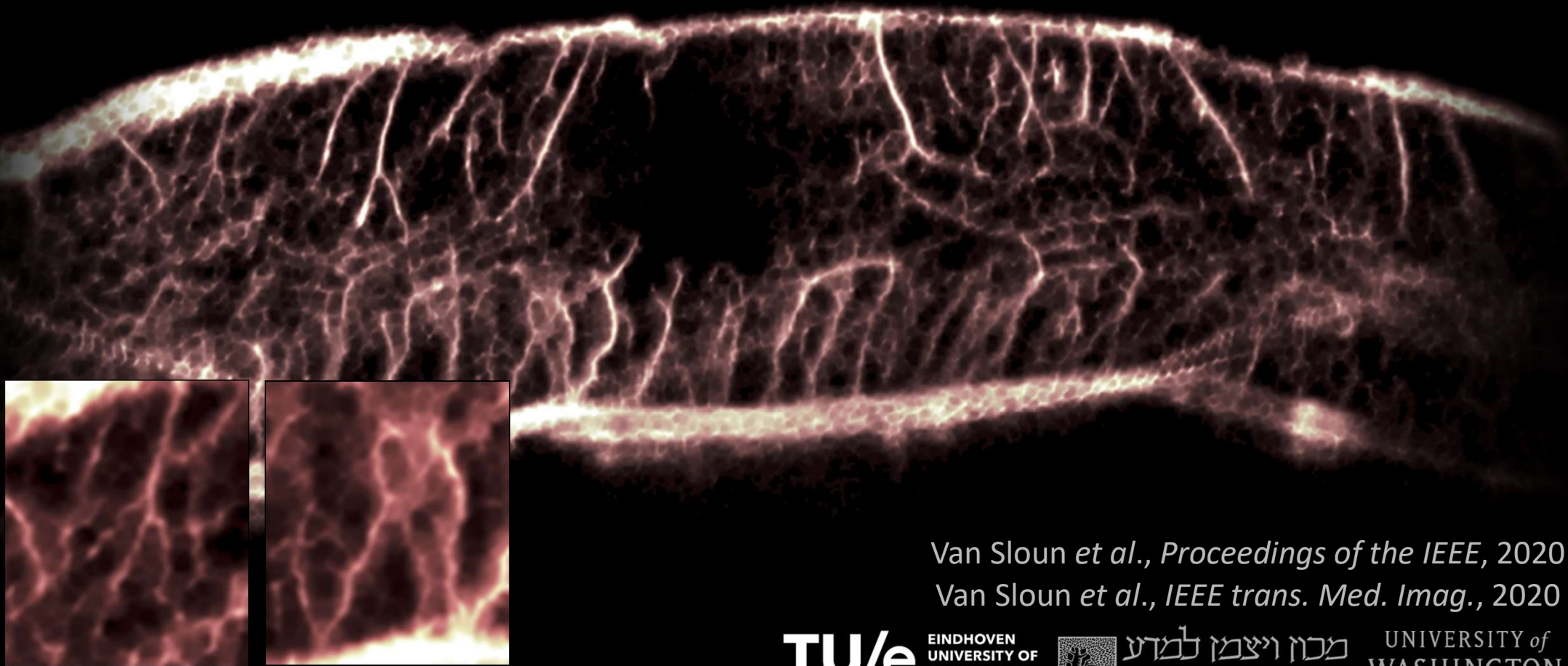
Deep unfolding



Standard contrast-ultrasound



Super-resolution contrast ultrasound by LISTA



Van Sloun *et al.*, *Proceedings of the IEEE*, 2020

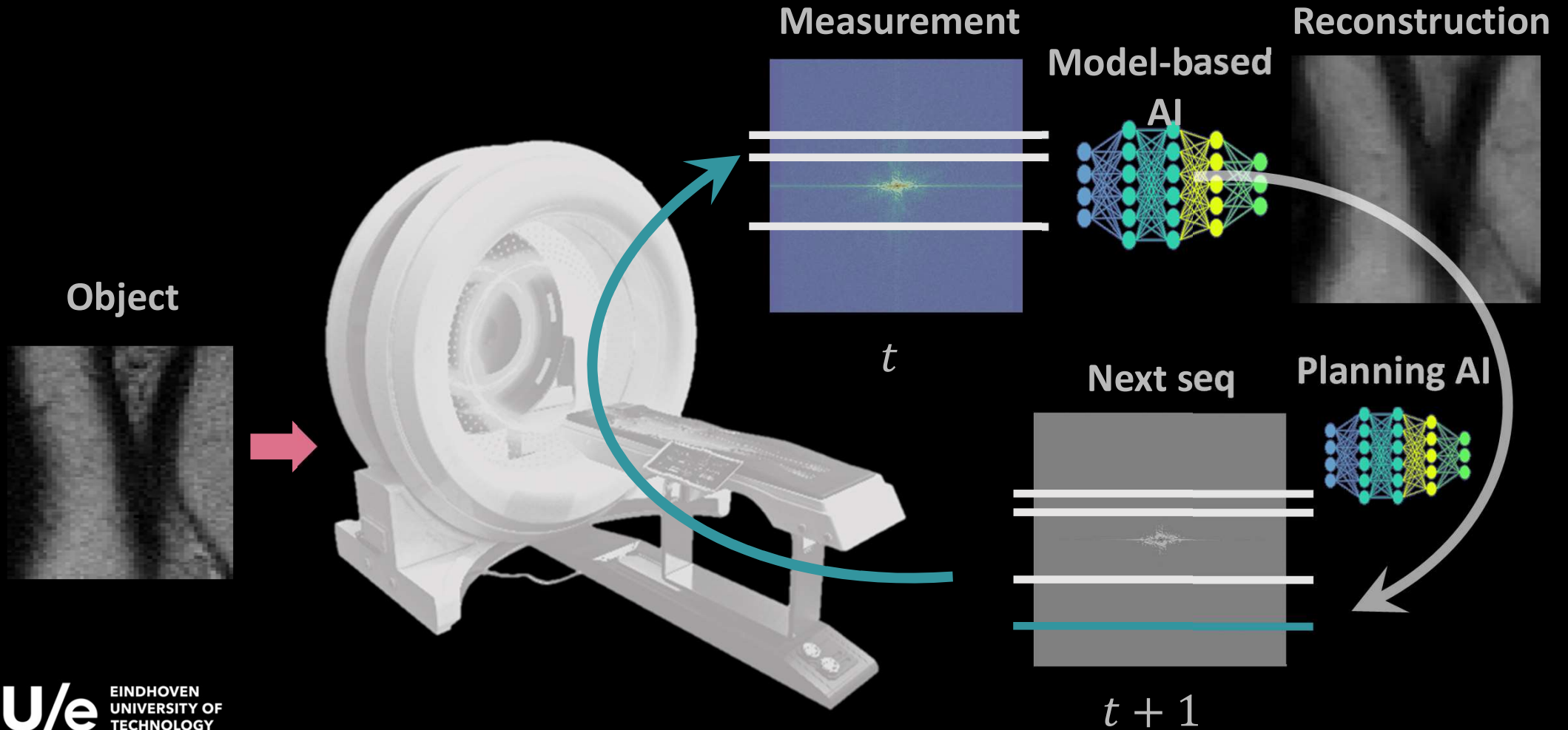
Van Sloun *et al.*, *IEEE trans. Med. Imag.*, 2020

Combining models, priors and deep learning for image reconstruction: use-cases



Magnetic Resonance Imaging
(faster, low-field etc.)

MRI: learning acquisition

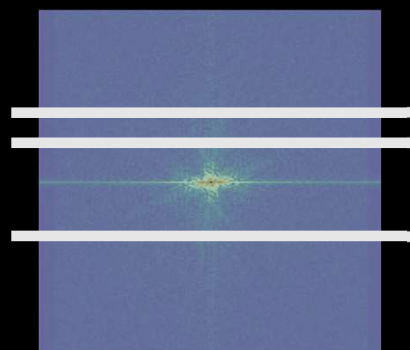


MRI: learning acquisition

Importance of model-based DL for recon:

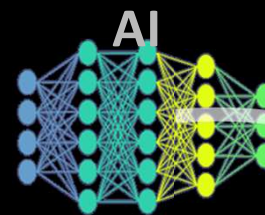
- Factorizes knowledge
- Sampling and recon update directions 'decoupled' during learning

Measurement

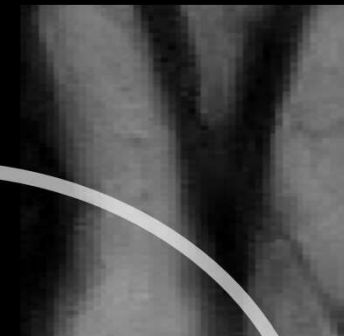


t

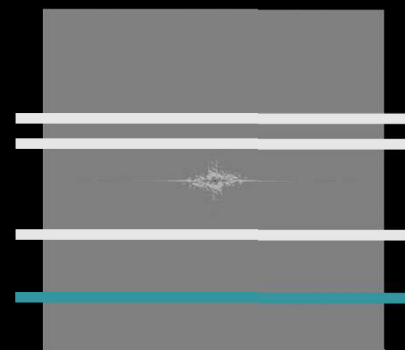
Model-based AI



Reconstruction

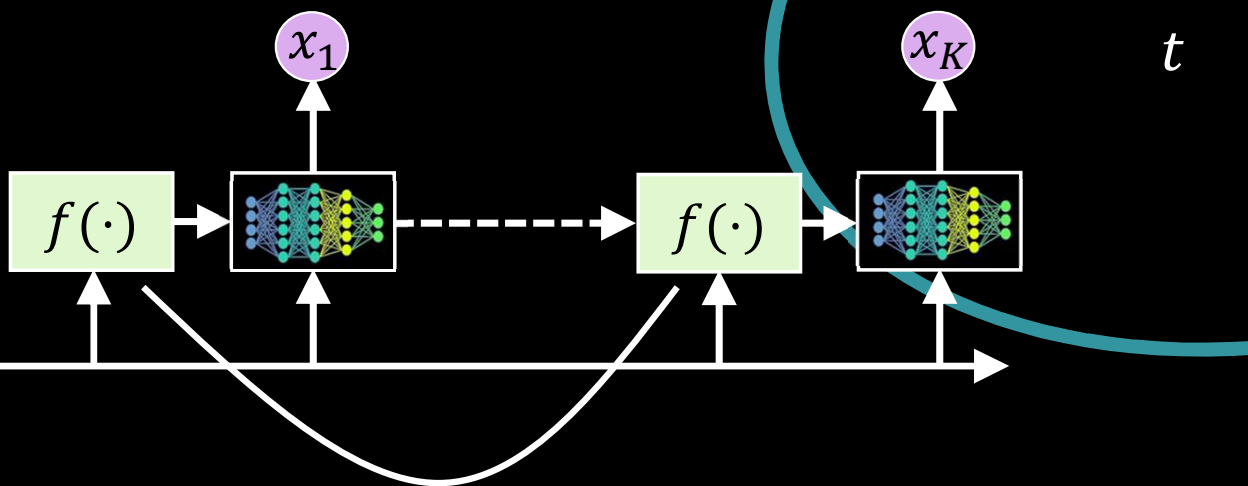
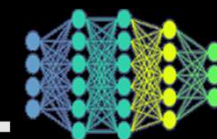


Next seq



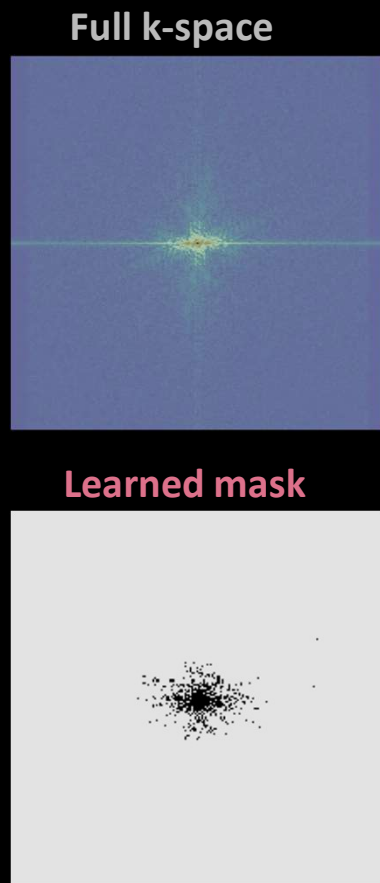
$t + 1$

Planning AI

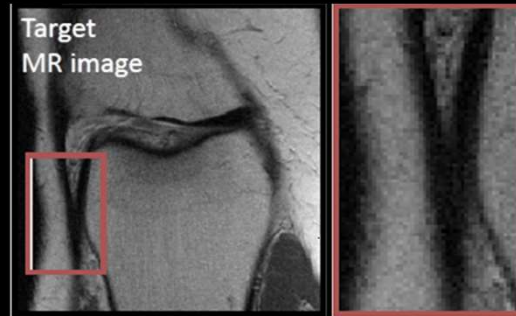


Acquisition/sampling changes directly update $f(\cdot)$

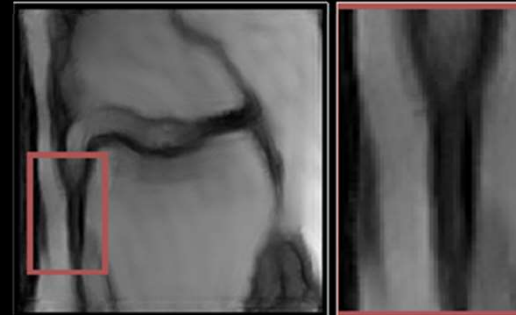
MRI: learning sampling density mask



Full sampling

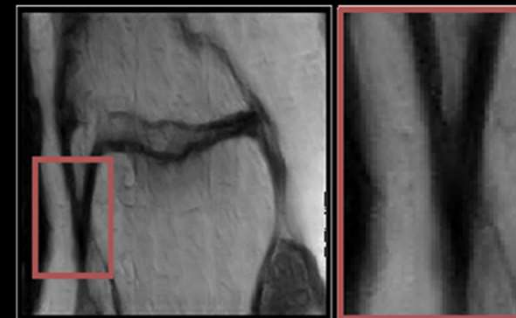


Sampling
Low frequencies



PSNR: 35.8

Learned sampling

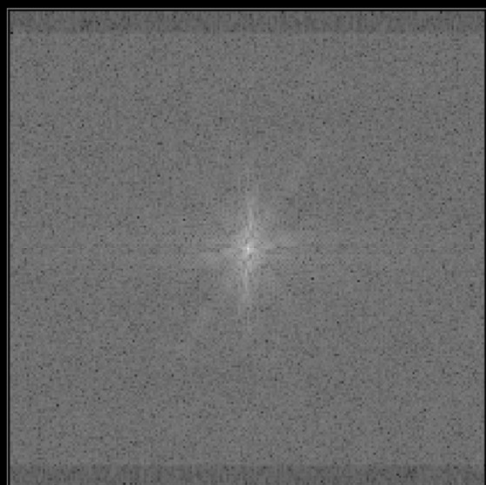


PSNR: 36.2

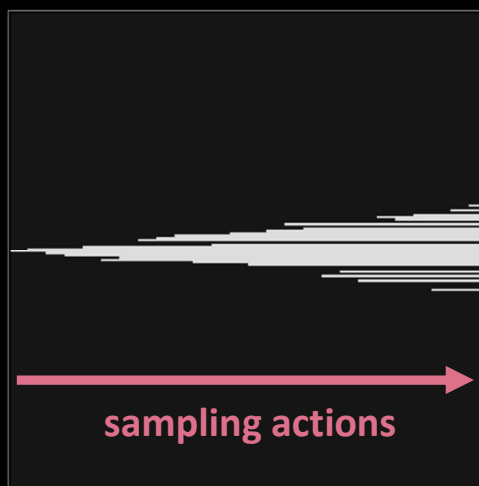
MRI: learning active acquisition

Factor 8 undersampling

K-space



Active line sampling
(cumulative)



Reconstruction

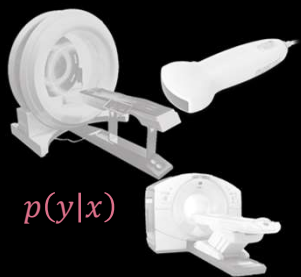


Target (full acquisition)



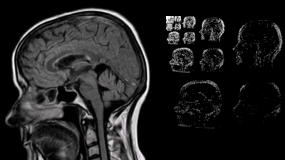
Method	NMSE	PSNR [dB]	SSIM
Zhang et al., 2019 (active)	0.0398	28.8	0.610
Pineda et al., 2020 (active)	0.0371	29.2	0.623
Fixed learned sampling (ours)	0.0360	30.1	0.650
Active acquisition (ours)	0.0342	30.2	0.654

Conclusions



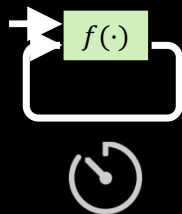
$p(y|x)$

Acquisition model assumptions incorrect/imprecise

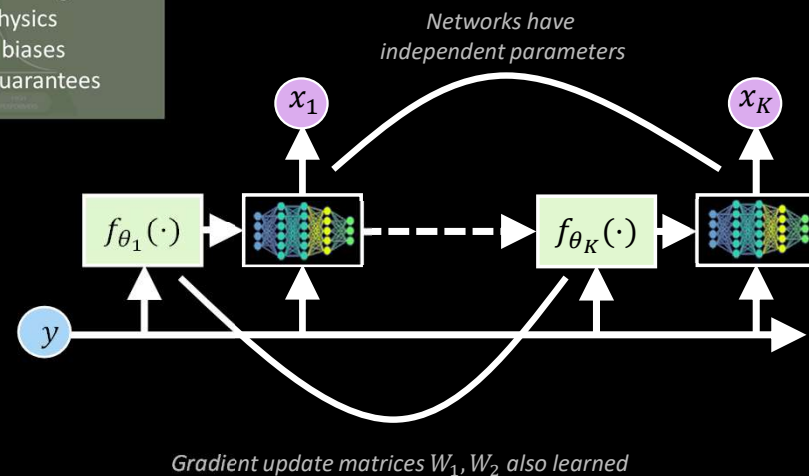
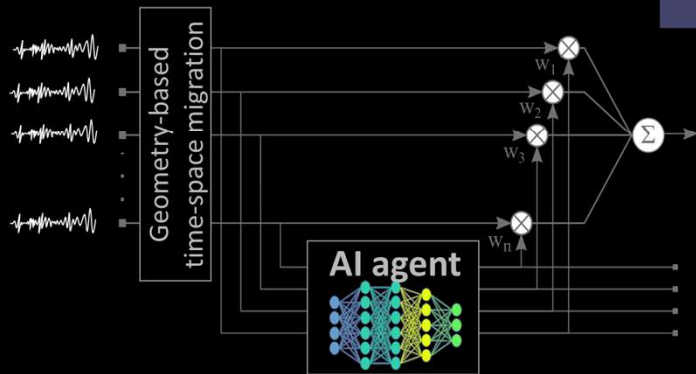
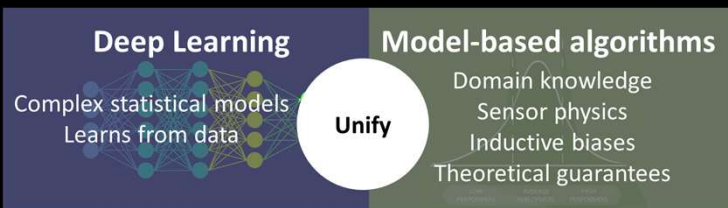
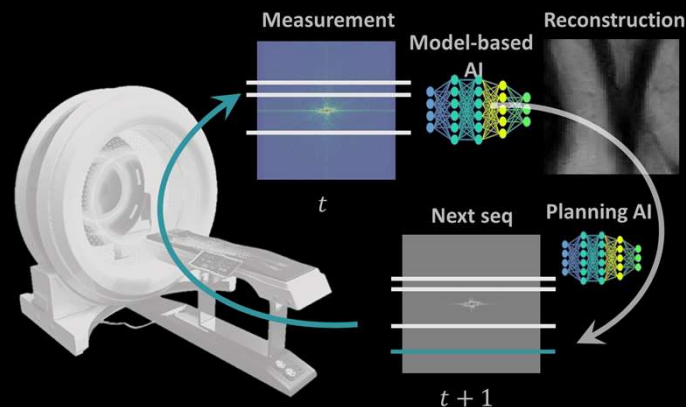


$p(x)$

Statistical image priors not sufficiently expressive/accurate



Slow reconstruction with high complexity



Thanks!